Searching for evidence: less can be more*

Florian Gössl[a] David J. Kusterer[a,†] Achim Wambach[b]

[a] *University of Cologne, Germany*

[b] *ZEW Mannheim, Germany*

July 26, 2019

We analyze a situation in which an uninformed decision-maker has to decide on an issue. There are two parties with state-independent opposing interests who can acquire information in support of their cause through costly search. Information can be obtained across multiple dimensions. A decision is called more complex the more dimensions are available for investigation. Each party has to decide on the number of searches it performs. If there is an asymmetry between the parties with regard to the utility they derive from decisions in their favor, we show that a reduction of complexity in the sense that admissible dimensions are restricted can lead to an overall increased and more balanced search, which can in turn improve welfare.

Keywords: information acquisition; informational lobbying; antitrust; competition policy

JEL classification: D83; K21; L4; L51

*We would like to thank Nicolas Fugger, Philippe Gillen, Vitali Gretschko, Oliver Gürtler, Wanda Mimra, Johannes Münster, Alexander Rasch, and Patrick Schmitz for helpful comments and discussions.

†Corresponding author. Email address: kusterer@uni-koeln.de. Postal address: Department of Economics, University of Cologne, Albertus-Magnus-Platz, 50923 Köln, Germany.

1 Introduction

Regulation and antitrust have become more complex. In financial regulation in the United States, the Dodd-Frank Act, which was signed into law in 2010 as a response to the 2008 financial crisis, is “23 times longer than Glass-Steagall” (Economist, 2012), the legislation passed in the 1930s as a response to the 1929 crash of Wall Street. In its execution, until 2014, 27,669 new “regulatory restrictions” have been put into place, while 30% of the Dodd-Frank Act rules have yet to be implemented (Economist, 2016). In the European Union, the European Commission (EC) observes increased complexity in merger cases. The EC states: “The recent trend that transactions become more complex has continued in 2013. Second phase investigations in particular generally require sophisticated quantitative and qualitative analyses involving large amounts of data.” (European Commission, 2014, p. 25). Baye and Wright (2011) show that decisions made by judges who did not receive basic economic training are more likely to be appealed, supporting the hypothesis that some antitrust cases have become too complicated for generalist judges.

Complexity itself may not be problematic, but it becomes an issue if the firms and government agencies involved in regulation and antitrust cases cannot adjust to it in a similar fashion. While firms presumably can easily increase their budget for legal and/or economic advice if deemed necessary, government agencies face binding budget constraints and may be unable to increase their workforce or keep enough competent staff on their payroll (see OECD, 2012). This asymmetry may lead to biased decisions and welfare losses, or, as Rogoff (2012) puts it for the case of financial regulation: “The problem, at least, is simple: As finance has become more complicated, regulators have tried to keep up by adopting ever more complicated rules. It is an arms race that underfunded government agencies have no chance to win.”

In a setting where an uninformed decision-maker has to approve or reject a proposal for which she has to rely on two biased parties that can search for multiple pieces of information and submit them to her, we find that reducing complexity may increase search activity and welfare if the two parties are asymmetric in the utility they derive from a decision for their respective cause.

The decision-maker in our model could correspond to a judge deciding on an antitrust or regulation case in the US or to a judge presiding over a case of white-collar crime. In the EU, the EC has the hybrid role of a biased party *and* the decision-maker. On the one hand, its goal is to protect consumer interests, on the other hand, it decides on whether to allow or block a merger. Because 'wrong' decisions can be reviewed and overturned by the European Court of Justice, we argue that our setting also applies to the European case.[1]

We assume that the two parties receive decision-based rewards in the sense of Dewatripont and Tirole (1999), that is, they only get rewarded if the decision is made in their respective favor: one party prefers approval, while the other prefers rejection. Applied to our initial example, one party may be interpreted as a firm filing for a merger with one of its competitors while the other party is the antitrust authority. In this situation, we argue that it is natural to assume that both parties benefit from a decision in their respective favor only and that the benefit of a cleared merger to the involved firm(s) by far outweighs the benefit of the (bureaucrats of the) antitrust authority in case of a blocked merger, which are mainly immaterial in the form of career concerns: The OECD (2005, p. 175) states that performance-based pay for civil servants, if in place at all, "[...] usually represent less than 10% maximum of the base salary" and also shares our view about the relevance of career concerns: "Job content and career development prospects have been found to be the strongest incentives for public employees." (ibid., p. 177). Although the use of performance pay in the public sector has increased in recent years, financial pressure due to the 2008 financial crisis has slowed down this trend: "Since 2008, nine OECD countries have reduced bonuses, allowances and performance-related pay." (OECD, 2015, p. 108). In contrast, investment bankers and lawyers in M&A cases receive high-powered incentives (see e.g. Dropkin, 2015).

Both parties can simultaneously search for information on multiple dimensions. We interpret the number of admissible dimensions as the complexity of a case. As an example, a dimension represents a specific analysis that has to be carried out in order to document the potential effects of a merger. The legislative can adjust complexity by requiring a larger or smaller number of analyses. If the decision-based reward of the disadvantaged party is too low to engage in any search for information initially, we show in a first step that reducing complexity, that is, decreasing the number of dimensions allowed for investigation, may increase search incentives of

[1] For a confirmation of this view from a juridical perspective, see Vesterdorf (2005).

this party holding constant full search by the other party. The reduction of complexity reduces the advantage of the privileged party which makes search more attractive for the disadvantaged party.

The decision-maker aims to maximize welfare but is neither informed about the state nor is she able to observe the search activity by the two parties. In a first-best world, the decision-maker is fully informed and hence does not generate welfare losses by wrong decisions. This could be reached in equilibrium if both parties search on all dimensions. In an equilibrium where one of the parties does not search on all dimensions, however, the decision-maker is not fully informed and cannot avoid decision errors. A reduction of complexity thus has two effects: it makes it impossible to reach the first-best but it can lead to increased search activity by the disadvantaged party which translates into more and more balanced information available to the decision-maker. We show in a second step that lowering the quantity of required information gathering can lead to an increase of welfare.

Our results suggest that it may be beneficial for welfare to simplify procedures in competition and regulation cases if the involved agents are asymmetric. This finding is consistent with the Regulatory Fitness and Performance program (REFIT) initiated by the EU which, regarding merger review, aims "to make the EU merger review procedures simpler and lighter for stakeholders and to save costs." (European Commission, 2014, p. 24)

Related literature Another application of our model is informational lobbying with competing interest groups. Policy-makers who have to decide on whether to vote in favor of or against new legislation are potentially uninformed about the implications of the new legislation but can rely on lobby groups to feed them with (possibly biased) information. Lobby groups benefit from a policy change in their favor and can invest resources to search for arguments and information supporting their preferred outcome. If such information is discovered, the group has an incentive to inform the policy-maker about it. Examples where the benefit of a favorable decision may differ significantly between interest groups include tobacco companies competing with consumer protection groups in order to avoid sales and/or marketing restrictions or oil companies lobbying for drilling rights or the legalization of fracking against environmental protection interest groups.

There are two different channels through which interest groups can influence the political decision process: campaign contributions and informational lobbying.[2] Interest groups can either supply politicians with information pertinent to the policy decision (Milgrom and Roberts, 1986; Austen-Smith and Wright, 1992; Potters and van Winden, 1992) or donate money to swing policy in their favor or help the preferred candidate to get elected (Prat, 2002a,b; Coate, 2004a,b), or both (Bennedsen and Feldmann, 2006; Dahm and Porteiro, 2008; Cotton, 2012). It is argued that informational lobbying is more prevalent, especially in the EU (Chalmers, 2013; New York Times, 2013), and more important compared to contributions (Potters and van Winden, 1992; Bennedsen and Feldmann, 2002). Generally, the literature on informational lobbying shows that decision-makers can learn something about the state of the world even from biased experts and improve policy by taking their information into account.[3] We show in our paper that with asymmetric lobby groups and multiple searches, the decision-maker may only receive information from the stronger group and welfare-reducing decision errors can occur. Simplifying the decision process by restricting the number of dimensions where information is taken into account for the decision results in more balanced information provision and increased welfare. A similar result has been found in the literature on contribution limits. Exertion of political influence by means of contributions is seen critical by the general public, which fears that wealthy groups can simply buy political favors (Prat, 2002b). In response, many countries use some form of contribution limits or try to reform campaign finance. Cotton (2012) analyzes a situation where a rich and a poor lobby group can pay contributions in order to get access to a decision-maker which is assumed to be essential for the transmission of information. In his model, contribution limits can be beneficial and yield more information transmission and better policy when interest groups can decide whether to form a lobby or not. Our paper is complementary to that literature in that it shows that welfare can be im-

[2] For an overview, see Grossman and Helpman (2001).

[3] Our model is also related to the more general literature on strategic information transmission started by Crawford and Sobel (1982). In these models, an uninformed decision maker (receiver) makes a decision based on information presented by one or more informed expert(s) (sender). The messages in these games typically are cheap talk while in our model, messages are verifiable, and senders can only send hard information they have gathered at a cost beforehand. A more recent overview of this literature is provided by Sobel (2013).

proved by simplifying the decision process when two asymmetric interest groups compete.

In our model, the interest groups are only interested in finding evidence in favor of their cause and hence are advocates in the sense of Dewatripont and Tirole (1999) who have shown that when agents receive decision-based rewards, competition between opposed agents can increase information gathering or render it cheaper for the principal (see also Austen-Smith and Wright, 1992).[4] Our information structure can be interpreted as an extension of the information structure of Dewatripont and Tirole (1999) to multiple dimensions. Similarly, but in a setting where messages are sent sequentially, Krishna and Morgan (2001) show that a decision-maker benefits from consulting two experts instead of one, but only when the experts' preferences are opposed. Shin (1998) compares an adversarial system, in which two opposed agents provide information, to an inquisitorial system, where an impartial decision-maker gathers information and finds the former to be optimal. This is due to the fact that the burden of proof can be allocated efficiently in an adversarial system when one agent is better informed than the other. Bennedsen and Feldmann (2006) look at the interplay of informational lobbying and contributions and find that if contributions are available, less information is transmitted in equilibrium. Competition between the groups cannot fully alleviate this result because an unsuccessful search creates an information externality which benefits the weaker group and thus decreases the search incentives of the stronger group.

Structurally, our model is also related to Bayesian persuasion (see e.g. Kamenica and Gentzkow, 2011). In this strand of literature, a sender wants to influence a receiver to take the sender's preferred action. Besides reporting a signal, the sender also designs the signal structure, i.e., the production of evidence. Gentzkow and Kamenica (2017) extend the model by introducing a competing second sender and provide a condition on the information environment such that competition leads to more information. Boleslavsky and Cotton (2016) use a similar setup with two senders and introduce limited capacity of the receiver to fund only one of two proposals. They show that this constraint leads to more information production compared to the unrestricted case. Our model is different in that one of the senders is

[4]See Hoppe and Kusterer (2011) for a related experimental study with conflicting tasks and agents as advocates.

constrained instead of the receiver. Furthermore, the signal structure is exogenously given and cannot be influenced by the agents.

The positive effect of reducing the action space of the agents has also been shown in the literature on optimal delegation (e.g. Szalay, 2005; Alonso and Matouschek, 2008; Armstrong and Vickers, 2010). In these models, a principal delegates decision-making authority to a self-interested agent. The principal has to decide how much liberty he wants to give to the agent. For example, in a model of interval delegation, Szalay (2005) shows that removing intermediate decisions from the agent's action set can improve his incentives for information gathering. In our model, the quality of decisions can be improved by restricting the information space through deliberate exclusion of one of the dimensions.[5]

Finally, our model is related to the contest literature.[6] Che and Gale (1998) show in an all-pay auction with asymmetric bidders that restricting the bid space by introducing a cap can increase competition and overall bids.[7] Because the cap can also lower the winning probability of the high value bidder, welfare may decrease. A difference to our setting is the definition of welfare. In Che and Gale (1998), welfare is maximized if the bidder with the largest valuation wins the auction while in our model, welfare is maximized if the correct decision conditional on the state of the world is taken, which is unrelated to valuation. Thus, for welfare maximization, both parties are ex-ante equally likely to win. Another difference to our setup lies in the relation between effort and winning. In the contest literature, players exert effort which translates into a probability of winning the contest through the contest success function. It is a standard assumption that the contest success function is at least weakly increasing in effort. In contrast, in our model the probability of winning may decrease in search activity due to the belief of the decision maker. In that sense, our paper is complementary to the literature on contests, showing a similar beneficial effect of leveling the playing field under different modeling assumptions.

The remainder of this paper is organized as follows. In Section 2 we present the model. The analysis of the game in Section 3 starts in Subsection 3.1 with the case where search is unrestricted and proceeds with the case where search on one

[5] Although not present in our model, agents could also bear cognitive costs of processing information. Lester et al. (2012) show that a judge can improve the accuracy of a jury's fact-finding by excluding evidence if the jury has to exert mental effort to process information.

[6] For a recent overview of the contest literature, see Konrad (2009).

[7] See Kirkegaard (2013) for a different setup in which one bidder is handicapped.

dimension is prohibited in Subsection 3.2. We then compare the search activity and the effects on welfare of the reduction in the number of dimensions in Subsection 3.3. A discussion follows in Section 4, and we conclude and provide an outlook in Section 5.

2 The model

A Bayesian judge (she) makes a decision $d \in \{-1, 1\}$ on a proposal. The judge can either accept ($d = 1$) or reject ($d = -1$) the proposal and is concerned with welfare maximization (see below). The sequence of events is summarized in Figure 1.

Welfare, and hence the decision, is based on ex ante unknown information on 3 dimensions, where a dimension is a specific analysis relevant to the case.[8] The information on each dimension $i \in \{1, 2, 3\}$ consists of the realization of two i.i.d. random variables $\theta_{i,j}$, $j \in \{f, r\}$.[9] Each $\theta_{i,j}$ takes value 1 with probability p and value 0 with probability $1 - p$, where $0 < p < 1$. $\theta_{i,f} = 1$ can be interpreted as information in favor of the proposal while $\theta_{i,r} = 1$ can be interpreted as information against the proposal in dimension i.[10] $\theta_{i,j} = 0$, $j \in \{f, r\}$ means that there is no information available either in favor or against the proposal in dimension i. The state of the world is defined as $\Theta = \{\sum_i \theta_{i,f}, \sum_i \theta_{i,r}\}$. As all dimensions are equally important for the decision (see below), the number of pieces of existing information in directions f and r is sufficient to describe the state of the world.

Two interested parties, called the firm and the regulator, can search for information on each of the available dimensions, incurring marginal cost c for a search effort on a dimension. Denote by $e_j \in \{0, 1, 2, 3\}$ the number of search efforts exerted by party $j \in \{f, r\}$. The firm prefers decision $d = 1$, while the regulator prefers $d = -1$, and both parties earn decision-based rewards in the sense of Dewatripont and Tirole (1999). The firm receives benefit $w_f \geq 0$ if $d = 1$, the regulator receives

[8] We argue that 3 is the smallest number of dimensions where restricting search can increase welfare. See our discussion in Section 4.

[9] This is a straightforward extension of the information structure in Dewatripont and Tirole (1999) to multiple dimensions.

[10] Given our definition, the probability that information exists is constant across dimensions i and directions f and r. An interpretation of this information structure is that it represents the residual uncertainty of a case.

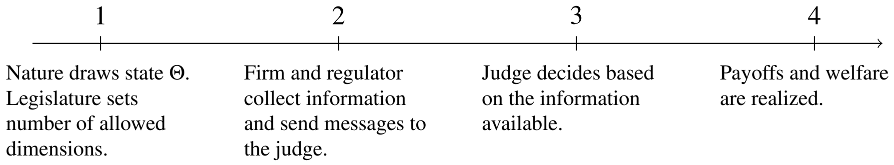

Figure 1: Sequence of events

benefit $w_r \geq 0$ if $d = -1$, and both receive zero otherwise.[11] Both parties maximize expected profits $u(w_j, d, c) = \Pr(d = \delta | e_f, e_r) w_j - e_j c$, where $\delta = 1$ ($\delta = -1$) for the firm (regulator). To account for asymmetry between the two parties, we assume $w_f > w_r$.[12] Furthermore we assume that the benefit accruing to the firm if the proposal is accepted is large enough such that full information collection on all dimensions always overcompensates the cost of doing so. It is easy to verify that there are values of w_f and c such that the firm prefers to search 3 times as long as $\Pr(d = 1 | e_f, e_r)$ is increasing in e_f, which holds as long as the judge believes that the firm conducts a full search on all dimensions. Hence, we assume that the firm always searches on all admissible dimensions such that it is no longer a strategic player in the game.

Welfare is given by $d\left[\sum_i \theta_{i,f} - \sum_i \theta_{i,r}\right]$ and depends on the decision and the state of the world.[13] Information is weighted equally across dimensions.[14] The judge's aim is to maximize expected welfare based on the information available to her. We assume that the decision-maker has no leeway and has to take the ex post optimal decision given the evidence presented to her. We believe that a judge or the legislature politically cannot implement a decision rule that is not welfare-optimal ex

[11] There are two interpretations of the benefit w_r for the regulator that are compatible with our model. It can be seen as capturing budgetary issues on the side of the regulatory agency, which is constrained in the resources it can expend for a given case. Our preferred view is that it captures the comparatively small private benefit of the bureaucrat working in the regulatory or competition agency, and that this benefit is mainly immaterial in the form of career concerns.

[12] The subscript on w is dropped later in the analysis whenever it is clear to which party w refers.

[13] We assume that the benefits w_j as well as the search costs c are insignificant relative to the positive (negative) societal welfare effect of a decision in favor of the party for which more (less) supportive information exists and hence omit them from our welfare definition. A similar welfare function is used in Cotton (2012) when abstaining from the possibility of monetary contributions to the decision maker he analyzes.

[14] Hence, depending on the sign of $\sum_i \theta_{i,f} - \sum_i \theta_{i,r}$ a merger is either good, bad, or irrelevant for society. Furthermore, mergers can be clearly better (or worse) than the status quo, or the decision could be on a knife-edge, depending on the size of the difference.

post.[15] Thus, in the case of full information, if there is (weakly) more information in favor of the proposal, i.e., $\sum_i \theta_{i,f} \geq \sum_i \theta_{i,r}$, it is optimal to accept the proposal and reject it otherwise. We assume that the proposal is accepted in case of a tie.[16]

In the situation we analyze there is incomplete information such that the state of the world is ex ante unknown. Without any information, the expected value of pro and contra information is the same in all dimensions and the judge accepts the proposal. In this situation, that decision reduces welfare whenever $\sum_i \theta_{i,r} > \sum_i \theta_{i,f}$. Hence, the judge is interested in gathering information to reduce expected welfare losses. She cannot search for information herself but has to rely on information made available to her by the firm and the regulator.

At the beginning of the game, the legislature chooses the number of dimensions which are relevant for the decision.[17] We assume that the firm is interested in searching information in favor of the proposal only, i.e., $\theta_{i,f}$, and that the regulator only searches for information against the proposal $\theta_{i,r}$. Both firm and regulator can simultaneously search in each dimension, and search effort is not observable. Denote the result of the search by party j in dimension i by $\hat{\theta}_{i,j}$. If a party searches in a given dimension i and there exists evidence in this dimension, it learns $\hat{\theta}_{i,j} = \theta_{i,j}$ with certainty. If a party does not search, it learns nothing, which we also denote by $\hat{\theta}_{i,j} = 0$. Hence, discovering that $\theta_{i,j} = 0$ and not searching for information yield the same result informationally, as in Dewatripont and Tirole (1999).

After searching, both parties $j \in \{f, r\}$ send a message $n_j = (\hat{\theta}_{1,j}, \hat{\theta}_{2,j}, \hat{\theta}_{3,j})$ to the judge. The judge summarizes this information as $m_j = \sum_{k=1}^{3} \theta_{k,j}$.[18] We assume that parties cannot withhold information. All discovered information is reported to the judge. This is natural in our case as parties only search for information that is beneficial to them and hence have no interest in holding it back.[19] The

[15]For similar arguments on limited commitment power, see Bennedsen and Feldmann (2006) or Boleslavsky and Cotton (2016).

[16]We argue that in case of a tie, there is no conclusive evidence against the proposal and thus, there is no obvious reason to decide against it. Our results do not change qualitatively if we use a tie breaking rule where the judge rejects the proposal or where she flips a fair coin.

[17]The legislature is a modeling shortcut of the political and societal processes that determine the laws relevant for competition or regulation cases.

[18]This captures the idea that the judge knows the dimension where a specific piece of information came from, which is necessary for her to be able to restrict the available dimensions.

[19]The only exception is the case where the judge believes that the regulator does not search. If the regulator does search and finds, say, one piece of information, he may prefer to withhold it. This hinges on the out-of-equilibrium belief of the judge. We argue that while the ability to

information available to the judge when maximizing expected welfare therefore is $M = \{m_f, m_r\}$, which we also call outcome.

The judge holds two types of beliefs. First, the judge has an expectation $\mu_j \in \{0,1,2,3\}$ about the number of searches of each party.[20,21] Second, she has a belief about the state of the world. Updating this belief not only depends on the messages received from the two parties but also on the expectation about the number of searches.

3 Analysis

We derive perfect Bayesian equilibria. An equilibrium consists of the number of searches by firm and regulator, the decision rule by the judge, and her beliefs μ_f and μ_r about the number of searches performed by the firm and regulator, respectively. We first analyze Situation 1, where searching in all three dimensions is allowed (the case of full complexity), and Situation 2, where the judge only accepts evidence from two of the three dimensions (reduced complexity).[22] All proofs are relegated to the Supplementary Material.

withhold information does increase the incentives to search for the regulator in this case, it does not qualitatively change our results.

[20] Technically, the belief of the judge is a probability distribution over the number of searches of each party, that is, a vector including 4 probabilities $\Pr(e_j = X)$ that party j's number of searches equals $X \in \{0,1,2,3\}$. In pure-strategy equilibria, the judge expects the parties to search a specific number of times such that one probability equals 1 and all other probabilities equal zero.

[21] In accordance with our assumption that the firm always searches on all available dimensions, we assume the judge's expectation about the number of searches by the firm, μ_f, to be equal to the number of admissible dimensions.

[22] Our analysis is only meaningful if the judge has access to information submitted to her by the parties only, ruling out the possibility that the judge might receive information from the prohibited dimension through other means. Given the confidential nature of most arguments in competition and regulation cases, publication or distribution via media outlets does not appear to be in the interest of the involved parties. This assumption is also common in the lobbying literature on access to legislators where information can only be submitted conditional on being granted access (see, for example, Austen-Smith (1998); Cotton (2009, 2012) and the references therein). As the legislative sets the number of admissible dimensions, another interpretation of the restriction of dimensions is that the judge is not allowed to take information on restricted dimensions into account when forming his decision, similar to evidence obtained illegally.

3.1 Situation 1: Information is accepted from all dimensions

3.1.1 Decision rules

We proceed backwards and first discuss the judge's decision rule for a given set of messages, M. The judge makes her decision based on the information submitted to her by the parties, m_f and m_r, and based on her beliefs about the number of searches of firm and regulator, e_f and e_r, in order to maximize expected welfare. Given our assumption that the firm always searches on all available dimensions, two situations are of particular interest for us: the case where the judge believes that the regulator also searches on all available dimensions as a benchmark and the case where the judge believes that the regulator does not search at all.[23] If the belief concerning the number of searches of party j is $\mu_j = 3$, it follows that the information submitted by j is believed to be the state of the world with probability 1. As a consequence, if the submitted information is, say, $n_j = (0,1,0)$ then $m_j = 1$, i.e. the judge believes that exactly one piece of information in favor of j exists.

Therefore, if the judge believes that both firm and regulator search three times ($\mu_f = \mu_r = 3$), the information available in equilibrium is equal to the state and she decides as in the case with full information according to the first-best decision rule: The decision is made in favor of the firm if it has found (weakly) more information than regulator and in favor of the regulator if it has found (strictly) more information. The optimal decision for all combinations of information $M = \{m_f, m_r\}$ for beliefs $\mu_f = \mu_r = 3$ is shown in Table 1.

		m_f = 0	1	2	3
m_r	0	F	F	F	F
	1	R	F	F	F
	2	R	R	F	F
	3	R	R	R	F

Table 1: Decision rule for $\mu_f = 3$, $\mu_r = 3$. F (R) denotes $d = 1$ ($d = -1$).

If the judge believes the regulator did not search ($\mu_r = 0$), there are only four possible equilibrium outcomes M after the firm has searched for information: $\{0,0\}$,

[23] The two other situations where the judge believes that the regulator searches once or twice are discussed in Supplementary Material B.

$\{1,0\}$, $\{2,0\}$, and $\{3,0\}$. If the regulator does not search, the judge does not learn any information against the proposal. Hence, she has to base her decision on the expected value of information existing in favor of the regulator, which is given by $3 \times p^3 + 2 \times 3(p^2(1-p)) + 1 \times 3p(1-p)^2 = 3p$. The optimal decision is determined by comparing the information submitted by the firm (which is equal to the state in equilibrium) with the expected value of information of the regulator.

Clearly, the proposal is rejected (accepted) if the firm finds no evidence (evidence on all dimensions). The optimal decision rule for the two intermediate cases depends on p. If the firm finds one piece of evidence and the proposal is accepted, expected welfare is given by $1 - 3p$ which is positive only for $p < 1/3$ and hence the judge will reject the proposal for values of p larger than $1/3$. Similarly, if the firm finds two pieces of evidence, expected welfare is given by $2 - 3p$.

We assume that if the judge receives a message other than 0 from the regulator when expecting him not to search, then she updates her (out-of-equilibrium) belief concerning the number of searches of the regulator to $\mu_r = 3$.[24] She updates her belief regarding the state such that the probability that it is equal to the message is equal to 1. It follows that the decision rule under full information applies out of equilibrium. The complete decision rule is given in Table 2.

			m_f = 0	1	2	3
m_r	0	$p \in (0, 1/3]$	R	F	F	F
	0	$p \in (1/3, 2/3]$	R	R	F	F
	0	$p \in (2/3, 1)$	R	R	R	F
	1		R	F	F	F
	2		R	R	F	F
	3		R	R	R	F

Table 2: Decision rule for $\mu_f = 3$, $\mu_r = 0$. F (R) denotes $d = 1$ ($d = -1$).

[24]As Bayes' rule does not apply in situations that occur with probability zero, there is no constraint on the out-of-equilibrium beliefs we specify. We chose the out-of-equilibrium belief that is least favorable for the regulator.

3.1.2 Search incentives

We first analyze the equilibrium candidate where the judge has the belief $\mu_r = 3$ (and, as we assume throughout, $\mu_f = 3$). The regulator anticipates the decision by the judge conditional on the information submitted to her. This gives rise to the following probabilities $\Pr(d = -1|e_r)$ of a decision against the proposal when he performs $e_r \in \{0, 1, 2, 3\}$ searches:[25]

$$\begin{aligned}
&\Pr(d = -1|0) = 0 \\
&\Pr(d = -1|1) = (1-p)^3 p \\
&\Pr(d = -1|2) = (1-p)^3(2p(1-p)+p^2)+3p(1-p)^2p^2 \\
&\Pr(d = -1|3) = (1-p)^3(1-(1-p)^3)+3p(1-p)^2(3p^2(1-p)+p^3)+3p^2(1-p)p^3.
\end{aligned}$$

For example, if the regulator searches one time, the decision is made in his favor if the firm does not find any information, which occurs with probability $(1-p)^3$, and if he finds one piece of information, which occurs with probability p. It is optimal for the regulator to search three times if the expected profit of searching three times is larger than the expected profit of searching twice (1), of searching once (2), and of not searching (3).

$$\Pr(d = -1|3)w - 3c \geq \Pr(d = -1|2)w - 2c \tag{1}$$

$$\Pr(d = -1|3)w - 3c \geq \Pr(d = -1|1)w - c \tag{2}$$

$$\Pr(d = -1|3)w - 3c \geq \Pr(d = -1|0)w \tag{3}$$

The following lemma gives a condition for the benefit of the regulator, w, under which these constraints hold such that the regulator matches the search effort by the firm.

Lemma 1. *There exists a critical value $\tilde{p} \in [0, 1]$ such that if and only if $w \geq \overline{w}$, where*

$$\overline{w} = \begin{cases} c/\left(p - 5p^2 + 16p^3 - 28p^4 + 26p^5 - 10p^6\right) & \text{for } 0 < p \leq \tilde{p} \\ c/\left(p - 4p^2 + \frac{28}{3}p^3 - 13p^4 + 10p^5 - \frac{10}{3}p^6\right) & \text{for } \tilde{p} < p \leq 1, \end{cases}$$

[25] As $e_f = 3$ we omit the reference to the number of the searches by the firm.

there exists an equilibrium in which the regulator and the firm search three times and the judge has beliefs $\mu_f = \mu_r = 3$.

The lemma defines a lower bound on the benefit w as a function of p above which a full search by both parties constitutes an equilibrium. The binding incentive constraint is determined by the smallest increase in the probability of a decision in favor of the regulator per unit of search cost when searching three times instead of two times, once or not at all.

When p tends to zero, the probability of winning tends to zero regardless of the number of searches. Because searching is costly, it can never be implemented. For small values of $p \leq \tilde{p}$, the marginal cost-adjusted increase in the probability of winning is smallest when moving from two to three searches and IC (1) is binding. The probability of a favorable decision is maximal for intermediate values of p, resulting in the lowest benefit necessary to implement full search. As p grows large ($p > \tilde{p}$), the probability of winning again tends to zero because it becomes more likely that the firm finds three pieces of information. Equation (3) becomes the binding IC because in this range of p, the marginal probability of winning is convex for the regulator and hence the average increase when moving from no search to a full search is smaller than the increase from one or two searches to three searches. A graphical illustration of the lower bound $\overline{w}$ is shown in Figure 2.

As a next step, we show under which conditions an equilibrium where the regulator does not search exists. If the judge has the belief $\mu_r = 0$, it is optimal for the regulator not to search if

$$\Pr(d = -1|0)w \geq \Pr(d = -1|3)w - 3c \tag{4}$$

$$\Pr(d = -1|0)w \geq \Pr(d = -1|2)w - 2c \tag{5}$$

$$\Pr(d = -1|0)w \geq \Pr(d = -1|1)w - c. \tag{6}$$

Clearly, these constraints can be satisfied by setting $w = 0$. The following lemma defines a threshold on w below which the regulator will not search for information.

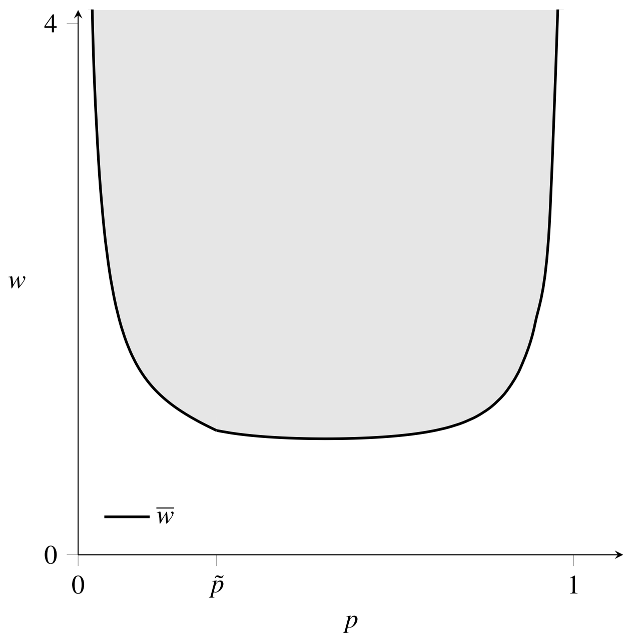

Figure 2: Lower bound for full-search equilibrium with $c = 1/10$. The grey area indicates the region where the full-search equilibrium exists.

Lemma 2. *There exists a critical value $\hat{p} \in [0,1]$ such that if and only if $w < \hat{w}$, where*

$$\hat{w} = \begin{cases} c/\left(3p^3 - 8p^4 + 8p^5 - 3p^6\right) & \text{for } 0 < p \leq \frac{1}{3} \\ \infty & \text{for } \frac{1}{3} < p \leq \hat{p} \\ -3c/\left(9p^2 - 36p^3 + 54p^4 - 39p^5 + 12p^6\right) & \text{for } \hat{p} < p \leq \frac{2}{3} \\ \infty & \text{for } \frac{2}{3} < p \leq 1, \end{cases}$$

there exists an equilibrium where the regulator does not search, the firm searches in all three dimensions, and the judge has beliefs $\mu_f = 3$ and $\mu_r = 0$.

Because the judge takes into account the information in favor of the regulator that may exist but is never discovered in equilibrium, searching only one or two times becomes increasingly unattractive for the regulator as p increases. Technically, the probability of winning decreases when searching once or twice instead of not searching. Additionally, searching is costly. Positive wages inducing a full search by the regulator exist only in regions of p just to the left of a change in the

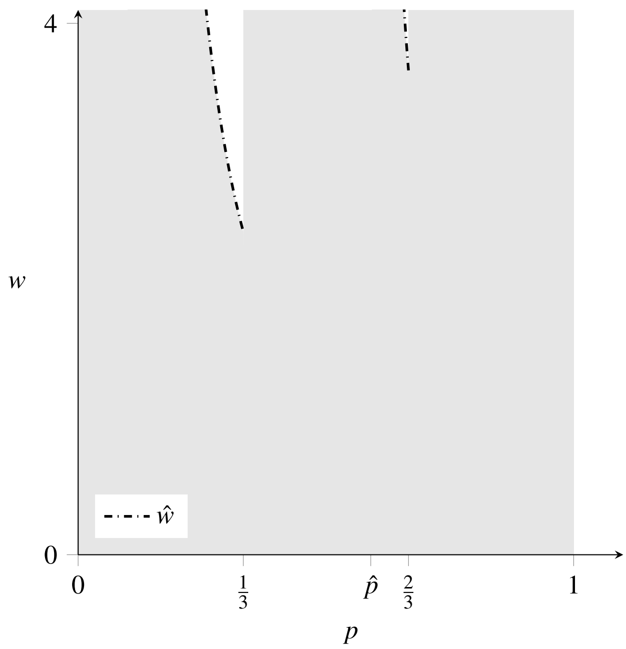

Figure 3: Upper bound for no-search equilibrium with $c = 1/10$. The grey area indicates the region where the no-search equilibrium exists.

decision rule. The area in which not searching is an equilibrium is colored in grey in Figure 3.

As we are interested in a situation where three searches by the regulator cannot be implemented, we next show that an equilibrium in which the regulator does not search exists for benefits below $\overline{w}$, i.e. the lowest possible benefit inducing three searches by the regulator, by comparing the two critical values $\overline{w}$ and $\hat{w}$.

Lemma 3. *It holds that $\hat{w} \geq \overline{w}$.*

An implication of the lemma is that the no-search equilibrium exists also for benefits below the minimum benefit necessary to implement full search. When the judge holds the belief that the regulator does not search, searching is relatively unattractive for the regulator because the judge takes into account the information that may exist but remains undiscovered. When the judge believes the regulator performs a full search, she will not decide in favor of the regulator if he does not deliver information, increasing the intrinsic motivation to search. A graphical comparison of the two critical values $\overline{w}$ and $\hat{w}$ is given in Figure 4.

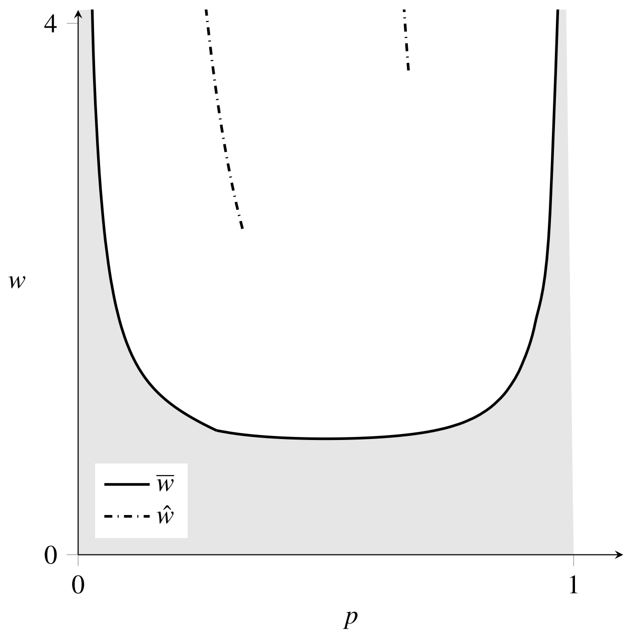

Figure 4: Comparison of thresholds for no-search and full-search equilibrium with $c = 1/10$. The grey area indicates the region where the full-search equilibrium does not exist while the no-search equilibrium exists.

To complete the analysis of pure-strategy equilibria under full complexity, we note that equilibria where the regulator searches one or two times also exist for benefits below $\overline{w}$. A full characterization of these equilibria can be found in the Supplementary Material B. In Lemma 4 we define constraints on w and p such that no search by the regulator is the unique equilibrium in pure strategies in Situation 1.

Lemma 4. *There exist critical values $\overline{w}_2$, $\tilde{w}$ and $\ddot{p}$ such that if and only if either (a) $w < \tilde{w}$ or (b) $\overline{w}_2 < w < \overline{w}$ and $1/2 < p < \ddot{p}$, no search by the regulator is the unique equilibrium given three searches by the firm.*

If the judge accepts evidence from all three available dimensions and if the benefit w for the regulator and the probability of evidence p are restricted in the way specified in Lemma 4, search activity is one-sided: the firm gathers evidence on all dimensions whereas the regulator does not search. The judge then only learns the arguments in favor of the firm but is not informed about arguments in favor of the regulator.

3.2 Situation 2: Restricted scope of information

In this section, we analyze the situation where the scope of information is restricted in the sense that the judge accepts information from two dimensions only. We characterize an equilibrium in which both parties search in all allowed dimensions. The probability that information exists in the third (unavailable) dimension is equal for both parties (and cases) and thus not relevant for the decision.

As in the case of full complexity, if the judge believes that both firm and regulator conduct a full search ($\mu_f = \mu_r = 2$) she takes the reported information to be equal to the true values and accepts the proposal if the firm has found (weakly) more information and rejects it otherwise. The judge's decision rule is displayed in Table 3.

		m_f = 0	1	2
	0	F	F	F
m_r	1	R	F	F
	2	R	R	F

Table 3: Decision rule for $\mu_f = 2$, $\mu_r = 2$. F (R) denotes $d = 1$ ($d = -1$).

The regulator has an incentive to search twice if

$$\Pr(d = -1|2)w - 2c \geq \Pr(d = -1|1)w - c \tag{7}$$

$$\Pr(d = -1|2)w - 2c \geq \Pr(d = -1|0)w. \tag{8}$$

The following lemma defines a range for the benefit w of the regulator in which an equilibrium where both regulator and firm search two times exists.

Lemma 5. *Suppose that search on one dimension is prohibited. If and only if $w \geq \underline{w}$, where*

$$\underline{w} = \begin{cases} c/\left(p - 3p^2 + 5p^3 - 3p^4\right) & \text{for } 0 < p < 1/3 \\ c/\left(p - \frac{5}{2}p^2 + 3p^3 - \frac{3}{2}p^4\right) & \text{for } 1/3 \leq p < 1, \end{cases}$$

there exists an equilibrium in which the regulator and the firm search on all two admissible dimensions and the judge has beliefs $\mu_f = \mu_r = 2$.

The shape of the lower bound on w defined in the lemma is qualitatively similar to $\overline{w}$ given in Lemma 1. As p tends to zero or one, the probability of winning tends to zero and a full search cannot be implemented. For small values of p the marginal increase in the probability of winning per unit of search cost is smallest for the second search while for large values of p, incentivizing full search as opposed to not searching at all leads to the lowest cost-weighted increase in the probability of a favorable decision. The lower bound $\underline{w}$ is displayed in Figure 5.

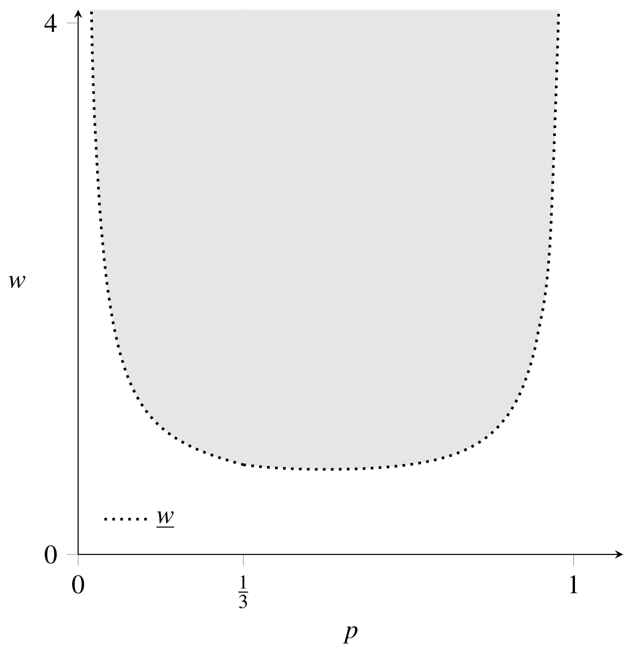

Figure 5: Lower bound for full-search equilibrium under reduced complexity with $c = 1/10$. The grey area indicates the region where the full-search equilibrium exists.

3.3 Comparison of Situations 1 and 2

3.3.1 Search activity

After solving the game separately in Situation 1, where search is unrestricted, and in Situation 2, where evidence on one dimension is not accepted by the judge, we now combine and summarize our previous results regarding the regulator's search activity in the following proposition. To set the stage for our first main result, we start by comparing the minimum wages necessary to implement the full-search equilibrium in both situations.

Proposition 1. *The minimum benefit $\overline{w}$ necessary to render three efforts optimal for the regulator when the judge accepts evidence on three dimensions is always larger than the minimum benefit $\underline{w}$ necessary to make two efforts optimal when only two dimensions are allowed.*

The proposition says that a range of benefits w exists where a full search by the regulator cannot be implemented when search is unrestricted, while a full search is an equilibrium when the scope of search is restricted to two dimensions. In particular, in connection with Lemma 3, which states that not searching is an equilibrium for benefits below $\overline{w}$, if $\underline{w} < w < \overline{w}$, i.e. if full search cannot be implemented when all dimensions are available, reducing complexity can increase the regulator's search activity. The range of benefits where full search is an equilibrium in Situation 2 but not in Situation 1 is depicted by the grey area in Figure 6.

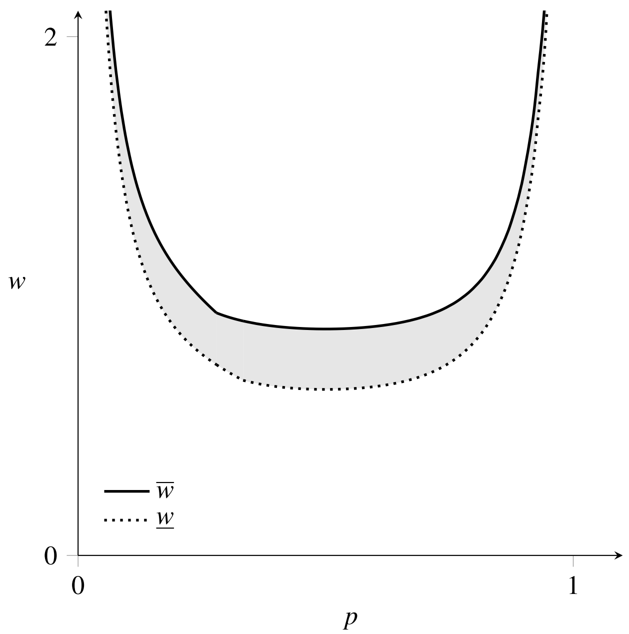

Figure 6: Comparison of lower bounds for full-search equilibria under full and reduced complexity with $c = 1/10$. The grey area indicates the region where the full-search equilibrium exists under reduced complexity but does not exist under full complexity.

Reducing the number of available dimensions increases the change in the probability of winning per unit of search costs in the binding incentive constraints, therefore decreasing the lowest benefit necessary to implement full search. If it is unlikely that information exists (small p), in both situations the binding constraint is given by comparing the expected profit from full search with the expected profit of searching on one dimension less than available. With reduced complexity, the cost-weighted increase in the winning probability is larger because the second search is more likely to be decisive. For large probabilities that information exists, the relevant comparison is between full search and no search. The increase in the probability of a decision in favor of the regulator when moving from no search to full search is larger under full complexity than under reduced complexity. When taking into account the cost of searching, however, reducing complexity leads to an increase in the probability of winning per unit of search costs. For both constraints, the necessary benefit to induce full search by the regulator is therefore smaller compared to the case of full complexity.

Taken together, if the benefit of the regulator is bounded from above, particularly if the benefits are below $\overline{w}$, he will not search if the scope of search is unrestricted. Prohibiting to search for information on one of the three dimensions lowers the benefit that is necessary to make the regulator willing to search on all available dimensions. This levels the playing field such that the regulator is able to search on as many dimensions as the firm.

If $w < \overline{w}$ there are two other equilibria in Situation 1 where the regulator searches once and twice, respectively. The following proposition shows that further restrictions on w and p lead to an unambiguously one-sided search activity: Under the conditions specified in the following proposition, if search is unrestricted, the regulator will never search while he will conduct a full search if complexity is reduced.

Proposition 2. *There exist critical values $\overline{w}_2$ and $\ddot{p}$ such that if and only if either (a) $\underline{w} \leq w < \tilde{w}$ and $0 < p < 1/3$ or (b) $\max\{\overline{w}_2, \underline{w}\} < w < \overline{w}$ and $1/2 < p < \ddot{p}$, the regulator searches in two dimensions if the scope of search is limited to two dimensions while he does not search if searching on all three dimension is allowed.*

Next we show that besides potentially increasing the regulator's search activity, a reduction of the number of admissible dimensions can also be welfare-enhancing.

3.3.2 Welfare

We now determine the expected welfare losses L_k, where $k \in \{1,2\}$ refers to Situation 1 or 2, due to decision errors under incomplete information. These errors occur when the decision made by the judge given M (the outcome of the search process) does not match the optimal decision given Θ (the state of the world). For example, if the judge believes that the firm conducts a full search while the regulator does not search in Situation 1 ($\mu_f = 3$, $\mu_r = 0$) and receives the message $M = \{1,0\}$, he decides in favor of the firm if $p \leq 1/3$. In this scenario, a loss of $3-1=2$ occurs if the state is $\Theta = \{1,3\}$.

In Situation 1, where we focus on the equilibrium where the regulator does not search, welfare losses can occur in the two intermediate outcomes, that is, if the firm has found evidence in one or two dimensions ($M = \{1,0\}$ or $M = \{2,0\}$). The cases in which welfare is reduced depend on the level of p as the judge's decision rule also depends on it. Each loss can be separated into the probability of an outcome where a loss can occur (which we call error-prone messages) and the probability that a loss actually occurs, conditional on receiving an error-prone message.

For $0 < p < 1/3$, a wrong decision is made by the judge when the firm has found one piece of evidence but two or three pieces of evidence exist for the regulator, or when the firm has found two pieces but there are three pieces of evidence favoring the regulator's case. The expected losses in this range of p are given by

$$\begin{aligned} L_1\Big|_{p\in\left(0,\frac{1}{3}\right]} &= 3p(1-p)^2 \times \left(3p^2(1-p)(2-1) + p^3(3-1)\right) \\ &\quad + 3p^2(1-p) \times p^3(3-2). \end{aligned}$$

If $1/3 < p < 2/3$, the judge decides in favor of the regulator if the the firm has found up to one piece of evidence. This decision rule results in a welfare loss if there exist no pieces of evidence for the regulator. A loss also occurs when the firm has found two pieces and there exist three pieces of evidence on the side of the regulator. In this case, the expected losses are

$$\begin{aligned} L_1\Big|_{p\in\left(\frac{1}{3},\frac{2}{3}\right]} &= 3p(1-p)^2 \times (1-p)^3(1-0) \\ &\quad + 3p^2(1-p) \times p^3(3-2). \end{aligned}$$

For $2/3 < p < 1$, the decision is made in favor of the regulator when the firm has found up to two pieces of evidence. In this case, wrong decisions are made again in case of $M = \{1,0\}$, and when the firm has found two pieces but there exist zero or only one piece for the regulator. The expected losses are

$$\begin{aligned}L_1\Big|_{p\in\left(\frac{2}{3},1\right)} &= 3p(1-p)^2 \times (1-p)^3(1-0)\\ &\quad + 3p^2(1-p) \times \left((1-p)^3(2-0) + 3p(1-p)^2(2-1)\right).\end{aligned}$$

Simplifying leads to a total expected welfare loss in Situation 1 of

$$L_1 = \begin{cases} 9p^3 - 21p^4 + 18p^5 - 6p^6 & \text{for } 0 < p < 1/3 \\ 3p - 15p^2 + 30p^3 - 30p^4 + 18p^5 - 6p^6 & \text{for } 1/3 < p < 2/3 \\ 3p - 9p^2 + 15p^3 - 21p^4 + 18p^5 - 6p^6 & \text{for } 2/3 < p < 1. \end{cases}$$

In Situation 2, welfare losses can occur in all outcomes in which both parties have found the same amount of evidence, that is, $M \in \{\{0,0\},\{1,1\},\{2,2\}\}$. In these cases, it is possible that additional evidence in favor of the regulator but not in favor of the firm exists in the additional dimension but is not discovered. The expected losses in this situation are

$$\begin{aligned}L_2 =& (1-p)^4 \times (1-p)p(1-0)\\ &+ 4p^2(1-p)^2 \times (1-p)p(2-1)\\ &+ p^4 \times (1-p)p(3-2)\\ =& p - 5p^2 + 14p^3 - 22p^4 + 18p^5 - 6p^6.\end{aligned}$$

A comparison of L_1 and L_2 gives the following result.

Proposition 3. *Under the conditions of Proposition 2 and for $2-\sqrt{3} < p < \sqrt{3}-1$, the reduction of the number of dimensions from three to two is welfare-enhancing.*

The proposition gives a condition under which it can be beneficial from a welfare perspective to reduce the complexity of the case and disallow evidence from one dimension when (i) one of the two parties who can search for information is disadvantaged in the sense that its benefit from the decision is smaller than the other party's, and (ii) the probability that evidence exists in a given dimension and in a given direction is intermediate. A graphical comparison of the expected losses in Situations 1 and 2 is given in Figure 7. The following intuition can be summarized such that when the probability p of existing information is either very small or very large, the uncertainty about the state of the world is small, such that the outcome of the search is unsurprising most of the time. In Situation 1, where the judge knows all the information in favor of the case, using the expected value of the information against the proposal does not lead to significant decision errors. Conversely, in Situation 2, where the judge has no information about one dimension, decision errors are more likely. In this parameter range, reducing complexity is not beneficial. For intermediate value of p, however, the uncertainty about the state of the world is larger, and a more balanced amount of evidence is preferred.

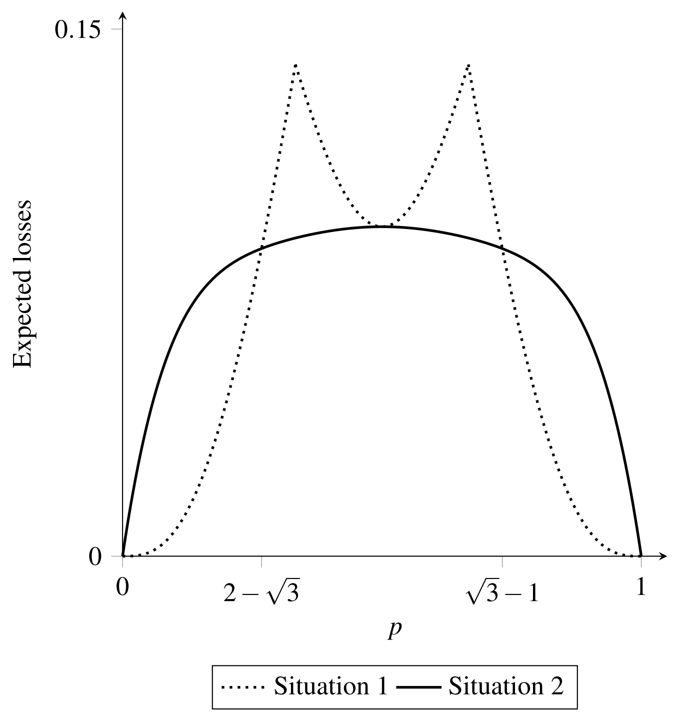

Figure 7: Expected welfare losses in Situation 1 (where search activity is asymmetric) and Situation 2 (where search is symmetric but information from one dimension is omitted).

For both small and large probabilities that information exists, the expected losses are smaller in a situation of asymmetric search activity. For small values of p, in

Situation 1, error-prone messages are very unlikely to occur because they require at least one piece of information. Conversely, in Situation 2, errors can occur if neither party finds any information ($M = \{0,0\}$), which is very likely for small p. Additionally, wrong decisions in Situation 1 occur if multiple pieces of information exist in favor of the regulator, which is also improbable. Conversely, in Situation 2 for errors to occur, only one additional piece of information is necessary, i.e., when there is information against the proposal but not in favor of it in the omitted third dimension, which is relatively likely. A similar argument can be made if the probability that information exists is large.

For intermediate probabilities that information exists, a situation of reduced complexity and symmetric search leads to lower expected losses. Two opposing effects are at work. On the one hand, the probability that an informational setting occurs in which wrong decisions can be made is large in Situation 1 and small in Situation 2, making losses more likely in the former situation; on the other hand the probability that a wrong decision is actually made is small in Situation 1 and large in Situation 2. We find that the first effect is stronger, resulting in larger expected losses when search is unrestricted but asymmetric. It is in this range of p where a reduction of complexity has an unambiguously welfare-enhancing effect.

4 Discussion

In this section we examine the robustness of our results. In particular, we discuss varying the number of initial dimensions as well as allowing the judge to ex ante commit to a decision rule.

Number of dimensions

Our model is based on three dimensions on which information can be found. We argue that this is the smallest number of dimensions where restricting search can increase welfare. The main difference between the two situations is that in Situation 1, if the regulator does not search, the judge learns all arguments in favor of the proposal but none against it, while in Situation 2, if search is restricted, she learns all pro and contra arguments on all but one dimensions. Moreover, in Situation 1, the judge adjusts the decision rule according to the expected value of information against the proposal, which can dampen welfare losses. In Situation 2, the expected value of information of the unavailable dimension is the same irrespective of the number of initial dimensions and does not affect the decision rule, and these losses cannot be avoided.

When there are two or less initial dimensions, knowing all evidence in one direction is better from a welfare perspective than not knowing any evidence from the excluded dimension. This is obvious if there is only one dimension, as restricting search in this case implies no search at all. For two initial dimensions, it is also easy to verify that the expected welfare losses with restricted search are strictly larger than under unrestricted search.[26] However, as the number of dimensions increases further, making the correct decision without information from the regulator becomes more difficult and the potential losses get larger, as the number of the intermediate outcomes where errors can occur – and thus the size of the errors – increases. In contrast, the losses when being fully informed about all but one dimensions decrease in comparison, as the single omitted dimension's relative impact on the decision becomes increasingly smaller. To put it simply – when the number of dimensions is n, then with full complexity, the overall number of searches will be n (only the firm searches), while with reduced complexity, the number of searches will be $2(n-1)$ (both parties search in one less dimension), and $2(n-1)$ is larger than n if $n \geq 3$. Thus it appears intuitive that the effect exists and may be even stronger for a larger number of dimensions.

So far we have assumed that the number of dimensions is exogenous. However, in actual cases, the firm could have some leeway in determining the number of dimensions, i.e., in choosing the degree of complexity. By hiring more lawyers and consultants, the firm might be able to file a larger report to the authorities. In this situation, the number of dimensions is determined endogenously. It is then to be expected that the firm will choose this number taking into account the incentives by the regulator to put effort into search. That is, the firm might strategically choose the size of complexity in order to minimize the search effort of the regulator. Then, again, reducing the number of dimensions from which information is accepted may have a positive effect on the search activity of the regulator. We leave the formal proof of these claims as open questions for future research.

[26]Observe that reducing complexity in this case also does not increase overall search activity $e_r + e_f$.

Ex ante commitment to a decision rule

We assume throughout that the judge makes the ex-post-optimal decision given the information available to her. Alternatively, one could assume that the judge can commit herself to a decision rule before firm and regulator search for information. Two cases can be distinguished.

If the rational judge chooses to commit to the decision rule that maximizes her objective function ex post, ex ante commitment can be interpreted as an equilibrium selection device. The more interesting case arises if we assume that the judge commits ex ante to a decision rule which possibly violates ex post optimality. There is a considerable tension between inducing (optimal) search activity and welfare losses. The judge can easily construct decision rules that support a full search by the regulator as an equilibrium for a larger range of w than discussed in the main text. As an example, consider a decision rule specifying a decision in favor of the regulator if he finds one or more pieces of information independent of the information submitted by the firm. It is straightforward to show that the minimum benefit necessary to induce a full search in this case is lower than $\overline{w}$ as specified in Lemma 1. However, there is a trade-off between enhancing the incentives to search by an ex ante commitment and possibly reducing welfare. In the current example, losses may be large because the judge commits to decide in favor of the regulator even if he only finds one piece of information while the firm finds three pieces. Deriving the optimal trade-off is beyond the scope of this article and may be an interesting direction for future research.

5 Conclusion

In merger cases, information about the potential effects on consumer surplus is essential for the decision-making of competition authorities. Similarly, in lobbying cases, policy-makers need access to information in order to draft sensible legislation. We analyze a model where a decision-maker has to decide on a proposal based on information transmitted to her by two interested parties, the firm and the regulator. The firm prefers the proposal to be accepted while the regulator benefits from a rejection. The firm (regulator) only searches for information in favor (against) the proposal. Information is multidimensional in the sense that there is information in favor and against the proposal in several dimensions. A dimension can be thought of as a specific analysis relevant to the assessment of the merger's welfare effect. The basis of our analysis is the assumption that the regulator receives a smaller benefit from winning than the firm. This assumption captures that the regulator typically consists of bureaucrats with fixed wages, while the firm employs consultants and lawyers with incentive contracts to defend their case. We show that this asymmetry between the two parties can lead to biased search activity where the firm searches for more information than the regulator. In this case, the decision-maker has to decide based on biased arguments which were obtained from only one interested party. We suggest to reduce the complexity of the information-gathering process by reducing the number of dimensions that the decision maker takes into account in cases where the probability of finding information is intermediate. We show that this can enable the disadvantaged regulator to catch up with the firm's search efforts. In turn, the decision-maker is provided with additional and more balanced information which results in a welfare increase if the probability that information exists is neither too small nor too large.

At a first glance it is sensible to include as many relevant aspects as possible in merger cases or when new legislation is drafted. However, this aim might not be achievable when the parties who provide the decision-maker with information are very asymmetric, for example when small citizens' initiatives compete with large energy companies lobbying for fracking rights, or when a competition authority examines a proposed merger by companies with an (for practical purposes) unlimited budget for legal advice. In such cases it can be beneficial to reduce the complexity of the procedure in order to level the playing field. Our findings are in line with recent efforts by the EU as part of the REFIT programme which, regarding merger review, aims "to make the EU merger review procedures simpler and lighter for stakeholders and to save costs." (European Commission, 2014, p. 24)

More research on the topic of asymmetric parties and multidimensional information is needed. A natural next step would be to generalize the present model to a setting including a variable number of initial dimensions. This would make it possible to study the optimal reduction of complexity depending on the number of initial dimensions and the degree of asymmetry. In a similar vein, allowing the firm to strategically choose the number of relevant dimensions could lead to interesting new results explaining the observation of asymmetric search effort.

References

Alonso, R., Matouschek, N., 2008. Optimal delegation. Review of Economic Studies 75, 259–293.

Armstrong, M., Vickers, J., 2010. A model of delegated project choice. Econometrica 78, 213–244.

Austen-Smith, D., 1998. Allocating access for information and contributions. Journal of Law, Economics, & Organization 14, 277–303.

Austen-Smith, D., Wright, J.R., 1992. Competitive lobbying for a legislator's vote. Social Choice and Welfare 9, 229–257.

Baye, M.R., Wright, J.D., 2011. Is antitrust too complicated for generalist judges? the impact of economic complexity and judicial training on appeals. Journal of Law and Economics 54, 1–24.

Bennedsen, M., Feldmann, S.E., 2002. Lobbying legislatures. Journal of Political Economy 110, 919–946.

Bennedsen, M., Feldmann, S.E., 2006. Informational lobbying and political contributions. Journal of Public Economics 90, 631–656.

Boleslavsky, R., Cotton, C., 2016. Limited capacity in project selection: competition through evidence production. Economic Theory , 1–37.

Chalmers, A.W., 2013. Trading information for access: informational lobbying strategies and interest group access to the european union. Journal of European Public Policy 20, 39–58.

Che, Y.K., Gale, I.L., 1998. Caps on political lobbying. American Economic Review 88, 643–651.

Coate, S., 2004a. Pareto-improving campaign finance policy. American Economic Review 94, 628–655.

Coate, S., 2004b. Political competition with campaign contributions and informative advertising. Journal of the European Economic Association 2, 772–804.

Cotton, C., 2009. Should we tax or cap political contributions? A lobbying model with policy favors and access. Journal of Public Economics 93, 831–842.

Cotton, C., 2012. Pay-to-play politics: Informational lobbying and contribution limits when money buys access. Journal of Public Economics 96, 369–386.

Crawford, V.P., Sobel, J., 1982. Strategic information transmission. Econometrica 50, 1431–1451.

Dahm, M., Porteiro, N., 2008. Side effects of campaign finance reform. Journal of the European Economic Association 6, 1057–1077.

Dewatripont, M., Tirole, J., 1999. Advocates. Journal of Political Economy 107, 1–39.

Dropkin, A., 2015. Skin in the game: The promise of contingency- based m&a fees. Georgetown Law Journal 103, 1061–1088.

Economist, 2012. Over-regulated America. URL: `http://www.economist.com/node/21547789`.

Economist, 2016. American bankers look forward to a bonfire of financial rules. URL: `http://www.economist.com/news/finance-and-economics/21710806-presidential-election-may-signal-big-changes-way-finance`.

European Commission, 2014. Commission staff working document: Accompanying the document Report from Commission on Competition Policy 2013. COM(2014) 249 final.

Gentzkow, M., Kamenica, E., 2017. Competition in persuasion. Review of Economic Studies 84, 300–322.

Grossman, G.M., Helpman, E., 2001. Special Interest Politics. MIT Press, Cambridge, MA.

Hoppe, E.I., Kusterer, D.J., 2011. Conflicting tasks and moral hazard: Theory and experimental evidence. European Economic Review 55, 1094–1108.

Kamenica, E., Gentzkow, M., 2011. Bayesian persuasion. American Economic Review 101, 2590–2615.

Kirkegaard, R., 2013. Handicaps in incomplete information all-pay auctions with a diverse set of bidders. European Economic Review 64, 98–110.

Konrad, K.A., 2009. Strategy and Dynamics in Contests. LSE Perspectives in Economic Analysis, Oxford University Press, New York, NY.

Krishna, V., Morgan, J., 2001. A model of expertise. Quarterly Journal of Economics 116, 747–775.

Lester, B., Persico, N., Visschers, L., 2012. Information acquisition and the exclusion of evidence in trials. Journal of Law, Economics, and Organization 28, 163–182.

Milgrom, P., Roberts, J., 1986. Relying on the information of interested parties. RAND Journal of Economics 17, 18–32.

New York Times, 2013. Lobbying bonanza as firms try to influence european union. URL: `http://www.nytimes.com/2013/10/19/world/europe/lobbying-bonanza-as-firms-try-to-influence-european-union.html?_r=0`.

OECD, 2005. Modernising Government: The Way Forwar. OECD Publishing, Paris.

OECD, 2012. Public Sector Compensation in Times of Austerity. OECD Publishing, Paris.

OECD, 2015. Government at a Glance 2015. OECD Publishing, Paris.

Potters, J., van Winden, F., 1992. Lobbying and asymmetric information. Public Choice 74, 269–292.

Prat, A., 2002a. Campaign advertising and voter welfare. Review of Economic Studies 69, 999–1017.

Prat, A., 2002b. Campaign spending with office-seeking politicians, rational voters, and multiple lobbies. Journal of Economic Theory 103, 162–189.

Rogoff, K., 2012. Ending the financial arms race. Project Syndicate. URL: `http://www.project-syndicate.org/commentary/ending-the-financial-arms-race-by-kenneth-rogoff`.

Shin, H.S., 1998. Adversarial and inquisitorial procedures in arbitration. RAND Journal of Economics 29, 378–405.

Sobel, J., 2013. Giving and receiving advice, in: Acemoglu, D., Arellano, M., Dekel, E. (Eds.), Advances in Economics and Econometrics. Cambridge University Press. volume 1. chapter 10, pp. 305–341.

Szalay, D., 2005. The economics of clear advice and extreme options. Review of Economic Studies 72, 1173–1198.

Vesterdorf, B., 2005. Standard of proof in merger cases: Reflections in the light of recent case law of the community courts. European Competition Journal 1, 3–33.

Supplementary Material – for online publication only

Supplementary Material A: Proofs

Proof of Lemma 1. For the moment, ignore (2). Rearranging (1) and (3) gives

$$\frac{c}{w} \leq p - 5p^2 + 16p^3 - 28p^4 + 26p^5 - 10p^6 \tag{9}$$

and

$$\frac{c}{w} \leq p - 4p^2 + \frac{28}{3}p^3 - 13p^4 + 10p^5 - \frac{10}{3}p^6, \tag{10}$$

respectively. The binding constraint is the stricter one, i.e., the one with the smaller RHS. Constraint (9) is binding if the RHS of (10) minus the RHS of (9) is positive, or $3 - 20p + 45p^2 - 48p^3 + 20p^4 \geq 0$. Define the LHS as $g(p)$. The third derivative $g'''(p) = -288 + 480p$ is increasing and crosses the abscissa once from below. Hence, $g''(p) = 90 - 288p + 240p^2$ is convex and has a local minimum at $p = 288/480$. As its value is positive at this local minimum, it is positive throughout the range of p. This implies that $g'(p) = -20 + 90p - 144p^2 + 80p^3$ has a positive slope, and it crosses the abscissa once from below as $g'(0) < 0$ and $g'(1) > 0$. Therefore, $g(p)$ is convex and has a local minimum, and because $g(0) > 0$ but $g(1) = 0$ and $g'(1) > 0$, its local minimum must be negative. Hence, $g(p)$ has a root, and it can be shown that it lies at

$$\tilde{p} = -\frac{1}{30}(586 + 45\sqrt{271})^{1/3} + \frac{59}{30(586 + 45\sqrt{271})^{1/3}} + \frac{7}{15} \approx 0.2794.$$

Hence, for $0 < p < \tilde{p}$, the condition above is satisfied and (1) is binding. For $\tilde{p} < p < 1$, (3) is the relevant constraint.

It remains to be shown that (2) is slack. Rearranging gives

$$\frac{c}{w} \leq p - \frac{9}{2}p^2 + \frac{25}{2}p^3 - 19p^4 + 15p^5 - 5p^6 \tag{11}$$

It suffices to show that the RHS of (11) is larger than the RHS (9), which is equivalent to $g_a(p) := 1 - 7p + 18p^2 - 22p^3 + 10p^4 \geq 0$ for $0 < p \leq \tilde{p}$ and larger than the RHS of (10) for $\tilde{p} < p < 1$, which is equivalent to $g_b(p) := -3 + 19p - 36p^2 + 30p^3 - 10p^4 \geq 0$.

The third derivative $g_a'''(p) = -132 + 240p$ is increasing and negative in the relevant range as $g_a'''(\tilde{p}) < 0$. It follows that $g_a''(p) = 36 - 132p + 120p^2$ has a negative slope and is positive as $g_a''(\tilde{p}) > 0$. Therefore, $g_a'(p) = -7 + 36p - 66p^2 + 40p^3$ is increasing and negative because $g_a'(\tilde{p}) < 0$. The original function $g_a(p)$ is decreasing but positive, as $g_a(\tilde{p}) > 0$. Hence, the condition above holds and (2) is slack for $0 < p < \tilde{p}$.

For $\tilde{p} < p < 1$, the third derivative $g_b'''(p) = 180 - 240p$ is decreasing and crosses the abscissa once from above because $g_b'''(\tilde{p}) > 0$ and $g_b'''(1) < 0$. The second derivative $g_b''(p) = -72 + 180p - 120p^2$ hence is concave and has a local maximum at $p = 3/4$. As it is negative at its local maximum it is negative throughout the relevant range of p and therefore $g_b'(p) = 19 - 72p + 90p^2 - 40p^3$ has a negative slope. From $g_b'(\tilde{p}) > 0$ and $g_b'(1) < 0$ follows that it crosses the abscissa once from above and that $g_b(p)$ is concave and has a local maximum in the range of interest. As $g_b(\tilde{p}) > 0$ and $g_b(1) = 0$ (and $g_b'(1) < 0$) we can infer that it is positive in the range of interest, the condition above holds and (2) is also slack for $\tilde{p} < p < 1$. $\square$

Proof of Lemma 2. The proof is divided in three parts according to the three ranges of p which differ in the decision rule—and hence in the probabilities of winning $\Pr(d = -1|e_r)$—as outlined in Table 2.

For $p \leq 1/3$,

$$\begin{aligned}
&\Pr(d = -1|0) = (1-p)^3 \\
&\Pr(d = -1|1) = (1-p)^3 \\
&\Pr(d = -1|2) = (1-p)^3 + 3p(1-p)^2 p^2 \\
&\Pr(d = -1|3) = (1-p)^3 + 3p(1-p)^2(3p^2(1-p) + p^3) + 3p^2(1-p)p^3.
\end{aligned}$$

Observe that (6) holds as long as $c \geq 0$, which is given by definition. Plugging in the relevant probabilities $\Pr(d = -1|e_r)$ in (4) and (5) and rearranging gives

$$\frac{c}{w} \geq 3p^3 - 8p^4 + 8p^5 - 3p^6 \tag{12}$$

and

$$\frac{c}{w} \geq \frac{3}{2}p^3 - 3p^4 + \frac{3}{2}p^5, \tag{13}$$

respectively. (4) is the binding constraint if the RHS of (12) is larger than the RHS of (13), which is equivalent to $\frac{3}{2} - 5p + \frac{13}{2}p^2 - 3p^3 > 0$. Define the LHS of this inequality as $g(p)$ where $g'(p) = -5 + 13p - 9p^2$. $g'(p)$ is strictly concave and takes a global maximum of $-11/36$ at $p = 13/18$. Hence $g(p)$ is decreasing, and with $g(0) > 0$ and $g(1) = 0$ we have shown that the sign of $g(p)$ is nonnegative in the relevant range of p. The wage in that range of p hence is given by (4), the incentive constraint preventing the regulator to conduct three instead of zero searches.

For $1/3 < p \leq 2/3$,

$$
\begin{aligned}
&\Pr(d = -1|0) = (1-p)^3 + 3p(1-p)^2 \\
&\Pr(d = -1|1) = (1-p)^3 + 3p(1-p)^2(1-p) \\
&\Pr(d = -1|2) = (1-p)^3 + 3p(1-p)^2((1-p)^2 + p^2) \\
&\Pr(d = -1|3) = (1-p)^3 + 3p(1-p)^2((1-p)^3 + 3p^2(1-p) + p^3) + 3p^2(1-p)p^3.
\end{aligned}
$$

We show that both incentive constraints (5) and (6) are always slack as the probability difference on the LHS is positive for all values of p. Observe that compared to zero searches, searching twice strictly reduces the winning probability and the regulator has to incur effort costs of $2c$. Searching twice hence can never be optimal. The same holds for searching once, which also never is optimal. It remains to be shown that (4) is slack for values of p below $\hat{p}$ and binding otherwise. Using the relevant winning probabilities in (4) and rearranging yields

$$\frac{c}{w} \geq -3p^2 + 12p^3 - 18p^4 + 13p^5 - 4p^6. \tag{14}$$

Define as $g(p) = -3 + 12p - 18p^2 + 13p^3 - 4p^4$, which is the RHS of (14) with p^2 factored out. The value of $g(0)$ is negative and the value of $g(1)$ is zero, so there can be at most three real roots in the range of p. We determine the actual number of roots between 0 and 1 by analyzing the derivatives of $g(p)$. $g'''(p) = 78 - 96p$ is a linear decreasing function with a positive value at $p = 0$ and a negative value at $p = 1$ and one root in between. Hence, $g''(p) = -36 + 78p - 48p^2$ is a concave function with one maximum in the relevant range. As $g''(p)$ is negative at the root of $g'''(p)$, its maximum, the second derivative of $g(p)$ is strictly negative in the domain from 0 to 1. Therefore, the first derivative $g'(p) = 12 - 36p + 39p^2 - 16p^3$

is decreasing in this range and has one root as $g'(0)$ is positive and $g'(1)$ is negative. Finally, this implies that $g(p)$ is concave and has a maximum in the domain from 0 to 1. Accordingly, there is one root at $p = \hat{p}$ where

$$\hat{p} = \frac{1}{4}3^{\frac{1}{3}} - \frac{1}{4}3^{\frac{2}{3}} + \frac{3}{4} \approx 0.5905$$

in that interval as the value of $g(1)$ is zero. Taken together, the RHS of (14) has roots at 0, $\hat{p}$, and 1, is negative for $0 < p < \hat{p}$ and positive for $\hat{p} < p < 1$. Since both c and w are positive, (14) is slack for $0 < p < \hat{p}$ and no positive wage induces the regulator to search three times. For $\hat{p} < p < 1$, this constraint is binding and yields the upper bound for the wage in the lemma.

For $p > 2/3$,

$$\begin{aligned}
\Pr(d = -1|0) &= (1-p)^3 + 3p(1-p)^2 + 3p^2(1-p) \\
\Pr(d = -1|1) &= (1-p)^3 + 3p(1-p)^2(1-p) + 3p^2(1-p)(1-p) \\
\Pr(d = -1|2) &= (1-p)^3 + 3p(1-p)^2((1-p)^2 + p^2) + 3p^2(1-p)(1-p)^2 \\
\Pr(d = -1|3) &= (1-p)^3 + 3p(1-p)^2((1-p)^3 + 3p^2(1-p) + p^3) \\
&\quad + 3p^2(1-p)((1-p)^3 + p^3).
\end{aligned}$$

The probability of winning when not exerting effort $\Pr(d = -1|0)$ consists of the probabilities that the firm finds zero, one, or two pieces of information. Searching once reduces the chances of winning because given one or two pieces of information found by the firm, the decision is now made against the proposal only if the regulator does not find information. A similar argument establishes that $\Pr(d = -1|2)$ and $\Pr(d = -1|3)$ are also smaller than $\Pr(d = -1|0)$. Hence, all constraints (4), (5), and (6) are slack and no positive wage induces the regulator to search for evidence. □

Proof of Lemma 3. We need to show that $\hat{w} \geq \overline{w}$. Several cases have to be considered. For $0 < p \leq \tilde{p}$, $\hat{w} \geq \overline{w}$ is equivalent to

$$\frac{c}{3p^3 - 8p^4 + 8p^5 - 3p^6} \geq \frac{c}{p - 5p^2 + 16p^3 - 28p^4 + 26p^5 - 10p^6}$$

or

$$1 - 5p + 13p^2 - 20p^3 + 18p^4 - 7p^5 \geq 0. \tag{15}$$

Define the LHS of (15) as $g(p)$. The fourth derivative $g''''(p) = 432 - 840p$ is decreasing and positive in the relevant range as $g''''(\tilde{p}) > 0$. Hence, $g'''(p) = -120 + 432p - 420p^2$ is increasing and negative as $g'''(\tilde{p}) < 0$. It follows that $g''(p) = 26 - 120p + 216p^2 - 140p^3$ has a negative slope and is positive throughout the range of interest as $g''(\tilde{p}) > 0$. The first derivative $g'(p) = -5 + 26p - 60p^2 + 72p^3 - 35p^4$ therefore increases but is negative as $g'(\tilde{p}) < 0$. From this we know that $g(p)$ is decreasing and positive as $g(\tilde{p}) > 0$ and the LHS of (15) is positive in the relevant range.

For $p > \tilde{p}$, the relevant comparison is

$$\frac{c}{3p^3 - 8p^4 + 8p^5 - 3p^6} \geq \frac{c}{p - 4p^2 + \frac{28}{3}p^3 - 13p^4 + 10p^5 - \frac{10}{3}p^6}$$

or

$$1 - 4p + (19/3)p^2 - 5p^3 + 2p^4 - (1/3)p^5 \geq 0. \tag{16}$$

Define the LHS of (16) as $g(p)$. The fourth derivative $g''''(p) = 48 - 40p$ is decreasing and positive. Hence, $g'''(p) = -30 + 48p - 20p^2$ is increasing and negative as $g'''(1) < 0$. Therefore, $g''(p) = 38/3 - 30p + 24p^2 - (20/3)p^3$ is decreasing and positive as $g''(1) = 0$. The first derivative $g'(p) = -4 + (38/3)p - 15p^2 + 8p^3 - (5/3)p^4$ hence is increasing and negative as $g'(1) = 0$. It follows that $g(p)$ is decreasing and positive as $g(1) = 0$, which implies that the LHS of (16) is positive in the relevant range.

For $1/3 < p \leq \hat{p}$, the regulator will not search for information for any positive value of w and hence, $\hat{w} \geq \overline{w}$ is always satisfied.

For $\hat{p} < p \leq 2/3$, the relevant comparison is

$$\frac{-3c}{9p^2 - 36p^3 + 54p^4 - 39p^5 + 12p^6} \geq \frac{c}{p - 4p^2 + \frac{28}{3}p^3 - 13p^4 + 10p^5 - \frac{10}{3}p^6}$$

or

$$1 - p - (8/3)p^2 + 5p^3 - 3p^4 + (2/3)p^5 \geq 0. \tag{17}$$

Define as $g(p)$ the LHS of (17). The fourth derivative $g''''(p) = -72 + 80p$ is increasing and negative in the relevant range as $g''''(2/3) < 0$. Hence, $g'''(p) = 30 - 72p + 40p^2$ has a negative slope and crosses the abscissa once as $g'''(\hat{p}) > 0$ and $g'''(2/3) < 0$. It follows that $g''(p) = -16/3 + 30p - 36p^2 + (40/3)p^3$ is concave with a local maximum. As both $g''(\hat{p})$ and $g''(2/3)$ are positive, the second derivative is positive throughout the range of interest. The first derivative $g'(p) = -1-(16/3)p+15p^2-12p^3+(10/3)p^4$ therefore is increasing and negative because $g'(2/3) < 0$. From this we know that $g(p)$ is decreasing and positive as $g(2/3) > 0$. This implies that inequality (17) strictly holds.

In the remaining interval $2/3 < p < 1$, the regulator will not search for any positive value of w and hence $\hat{w} \geq \overline{w}$ is satisfied. $\square$

Proof of Lemma 4. We show that $\overline{w}$ is either smaller than the lower bound or larger than the upper bound of the wage necessary for the other two equilibria where the regulator searches once or twice.

We start by showing that for $0 < p < 1/2$, where there are equilibria in which the regulator searches once or twice, $\underline{w}_1$ is smaller than $\underline{w}_2$ and hence is the relevant wage to be compared with $\overline{w}$ in order to determine $\min\{\overline{w}, \underline{w}_1, \underline{w}_2\}$. The comparison

$$\underline{w}_1 = \frac{c}{3p^2-6p^3+3p^4} \leq \frac{c}{3p^2-9p^3+12p^4-6p^5} = \underline{w}_2$$

can be simplified to $1-3p+2p^2 \geq 0$. The LHS is decreasing in the relevant range as the derivative $-3+4p$ is negative for $0 < p < 1/2$. At $1/2$, the LHS is zero, and hence, the condition above holds.

The relevant upper bound is given by $\overline{w}_2$ if

$$\overline{w}_2 = \frac{c}{3p^2-12p^3+24p^4-24p^5+9p^6} \geq \frac{c}{3p^2-9p^3+9p^4-3p^5} = \overline{w}_1$$

or $3p^3(1-5p+7p^2-3p^3) \geq 0$. Define the term in parentheses as $g(p)$. The second derivative $g''(p) = 14-18p$ is positive and decreasing in the relevant range as $g''(0.5)$ is positive. Hence, $g'(p) = -5+14p-9p^2$ is increasing in the negative domain because $g'(0.5)$ is negative. This implies that $g(p)$ is decreasing, and we know further that it must have one root as $g(0)$ is positive but $g(0.5)$ is negative. It can be shown that $g(p)$ crosses the abscissa at $p = 1/3$. Hence, for $0 < p < 1/3$, the relevant upper bound is given by $\overline{w}_2$ and by $\overline{w}_1$ for $1/3 < p < 1/2$.

For $0 < p \leq \tilde{p}$, we start by determining the relevant lower bound below which no search by the regulator is the unique equilibrium. The relevant comparison is

$$\overline{w} = \frac{c}{p - 5p^2 + 16p^3 - 28p^4 + 26p^5 - 10p^6} \leq \frac{c}{3p^2 - 6p^3 + 3p^4} = \underline{w}_1$$

or $p(1 - 8p + 22p^2 - 31p^3 + 26p^4 - 10p^5) \geq 0$. Define the term in parentheses as $g(p)$. The fourth derivative $g''''(p) = 624 - 1200p$ is positive in the relevant range $[0, \tilde{p}]$. Therefore, $g'''(p) = -186 + 624p - 600p^2$ is increasing in this range and negative for both $p = 0$ and $p = \tilde{p}$ and hence has no root. The second derivative $g''(p) = 44 - 186p + 312p^2 - 200p^3$ is strictly decreasing in that interval and takes on a positive value both at $p = 0$ and at $p = \tilde{p}$, and thus has no root in the relevant interval. The first derivative $g'(p) = -8 + 44p - 93p^2 + 104p^3 - 50p^4$ is negative for both $p = 0$ and $p = \tilde{p}$ and thus has no root as it is strictly increasing in that range of p. Hence, $g(p)$ is strictly decreasing in that interval and has one root as $g(0)$ is positive and $g(\tilde{p})$ is negative. It can be shown that the root lies at

$$\begin{aligned}\dot{p} = {} & \frac{2}{5} + \frac{1}{10\sqrt{\frac{6}{-54+5\left(486-27\sqrt{323}\right)^{1/3}+15\left(18+\sqrt{323}\right)^{1/3}}}} \\ & - \frac{1}{2}\Bigg[-\frac{18}{25} - \frac{1}{30}\left(486 - 27\sqrt{323}\right)^{1/3} - \frac{1}{10}\left(18 + \sqrt{323}\right)^{1/3} \\ & + \frac{3}{25}\sqrt{\frac{6}{-54 + 5\left(486 - 27\sqrt{323}\right)^{1/3} + 15\left(18 + \sqrt{323}\right)^{1/3}}}\Bigg]^{1/2} \approx 0.23802.\end{aligned}$$

It follows that $\overline{w}$ is smaller than $\underline{w}_1$ up to that root. This implies that the relevant lower bound is given by $\overline{w}$ from $p = 0$ up to the root and then by $\underline{w}_1$ until $p = \tilde{p}$.

Next, we show that $\overline{w}$ lies below the relevant upper bound in this range given by $\overline{w}_2$. The comparison

$$\overline{w} = \frac{c}{p - 5p^2 + 16p^3 - 28p^4 + 26p^5 - 10p^6} \leq \frac{c}{3p^2 - 12p^3 + 24p^4 - 24p^5 + 9p^6} = \overline{w}_2$$

can be simplified to $1 - 8p + 28p^2 - 52p^3 + 50p^4 - 19p^5 \geq 0$. Define the LHS of the last inequality as $g(p)$. The fourth derivative $g''''(p) = 1200 - 1140p$ is positive throughout the relevant range and hence, $g'''(p) = -312 + 1200p - 1140p^2$ is increasing. Observe that $g'''(\tilde{p})$ is negative which implies that the third derivative is negative throughout the range. Therefore, $g''(p) = 56 - 312p + 600p^2 - 380p^3$ is decreasing and positive, as $g''(\tilde{p})$ is positive. This indicates that $g'(p) = -8 + 56p - 156p^2 + 200p^3 - 95p^4$ increases in that range of p, and as $g'(\tilde{p})$ is negative, the first derivative is negative throughout. Taken together, we now know that $g(p)$ is decreasing, and as $g(\tilde{p})$ is positive, that it has no root in that range. The condition above holds.

For $\tilde{p} < p < 1/2$ we show first that

$$\underline{w}_1 = \frac{c}{3p^2 - 6p^3 + 3p^4} \leq \frac{c}{p - 4p^2 + \frac{28}{3}p^3 - 13p^4 + 10p^5 - \frac{10}{3}p^6} = \overline{w}$$

or $-3 + 21p - 46p^2 + 48p^3 - 30p^4 + 10p^5 \geq 0$. Define the LHS as $g(p)$. The fourth derivative $g''''(p) = -720 + 1200p$ is increasing and negative in the relevant range as $g''''(1/2) < 0$, implying a negative slope of $g'''(p) = 288 - 720p + 600p^2$. The positive value of $g'''(1/2)$ shows that the third derivative is positive throughout. Hence, $g''(p) = -92 + 288p - 360p^2 + 200p^3$ increases and is negative as $g''(1/2) < 0$, also implying that $g'(p) = 21 - 92p + 144p^2 - 120p^3 + 50p^4$ is decreasing. As $g'(\tilde{p}) > 0$ but $g'(1/2) < 0$ the first derivative has one root in the range of interest. The function $g(p)$ hence first is increasing and then decreasing, and because both $g(\tilde{p})$ and $g(1/2)$ are positive, it is positive throughout the range of interest and hence the condition above holds. Therefore, $\underline{w}_1$ is the relevant lower bound for this range.

Second, for the same range of p we show that

$$\overline{w} = \frac{c}{p - 4p^2 + \frac{28}{3}p^3 - 13p^4 + 10p^5 - \frac{10}{3}p^6} \leq \frac{c}{3p^2 - 9p^3 + 9p^4 - 3p^5} = \overline{w}_1$$

or $3 - 21p + 55p^2 - 66p^3 + 39p^4 - 10p^5 \geq 0$. Define the LHS as $g(p)$. The fourth derivative $g''''(p) = 936 - 1200p$ is decreasing in the positive domain, and hence $g'''(p) = -396 + 936p + -600p^2$ is increasing. Because $g'''(1/2) < 0$ the third derivative is negative throughout, implying a negative slope of $g''(p) = 110 - 396p + 468p^2 - 200p^3$. The second derivative is positive throughout as $g''(1/2) > 0$, and thus $g'(p) = -21 + 110p - 198p^2 + 156p^3 - 50p^4$ is increasing. Since $g'(\tilde{p}) < 0$ and $g'(1/2) > 0$, the first derivative has one root, and hence $g(p)$ is decreasing first and then increasing. It can be easily verified that $g(p)$ is positive at this root of $g'(p)$ and hence is positive throughout, satisfying the condition above. Therefore, there is no range of p and w below $\overline{w}$ where not searching is the unique equilibrium.

Taken together, we can now define

$$\tilde{w} = \begin{cases} \overline{w} & \text{for} \quad 0 < p < \dot{p} \\ \underline{w}_1 & \text{for} \quad \dot{p} < p < 1/2 \\ \underline{w}_2 & \text{for} \quad 1/2 < p < 1 \end{cases}$$

as used in the proposition.

For $1/2 < p < 1$, where the equilibrium in which the regulator searches twice also exists, we now show that the lower bound is given by $\underline{w}_2$ and that there also exists an area below $\overline{w}$ and above $\underline{w}_2$ where not searching is the unique equilibrium. The first condition is

$$\underline{w}_2 = \frac{c}{3p^2 - (15/2)p^3 + (15/2)p^4 - 3p^5} \leq \frac{c}{p - 4p^2 + \frac{28}{3}p^3 - 13p^4 + 10p^5 - \frac{10}{3}p^6} = \overline{w}$$

or $-12 + 84p - 202p^2 + 246p^3 - 156p^4 + 40p^5 \geq 0$. Define the LHS as $g(p)$. The fourth derivative $g''''(p) = -3744 + 4800p$ is increasing and crosses the abscissa once from below. This implies that $g'''(p) = 1476 - 3744p + 2400p^2$ first has a decreasing and then an increasing slope, and as it is positive at the root of $g''''(p)$, it is positive throughout the relevant range. From this fact we know that $g''(p) = -404 + 1476p - 1872p^2 + 800p^3$ is increasing, and it is negative and has no root in the relevant range as $g''(1) = 0$. Hence, $g'(p) = 84 - 404p + 738p^2 - 624p^3 + 200p^4$ is decreasing, and has one root as $g'(1/2) > 0$ but $g'(1) < 0$. The original function $g(p)$ thus has a local maximum at this root and is positive throughout as both $g(1/2)$ and $g(1)$ are positive, and the condition above is satisfied.

Lastly, we show that

$$\overline{w}_2 = \frac{c}{3p^2 - 12p^3 + 24p^4 - 24p^5 + 9p^6} \leq \frac{c}{p - 4p^2 + \frac{28}{3}p^3 - 13p^4 + 10p^5 - \frac{10}{3}p^6} = \overline{w}$$

or $-3 + 21p - 64p^2 + 111p^3 - 102p^4 + 37p^5 \geq 0$ for some values of p. Define the LHS as $g(p)$. The fourth derivative $g''''(p) = -2448 + 4440p$ crosses the abscissa once from below, implying a local minimum of $g'''(p) = 666 - 2448p + 2220p^2$. As $g'''(1/2) < 0$ and $g'''(1) > 0$, the third derivative has one root in the range of interest. Hence, $g''(p) = -128 + 666p - 1224p^2 + 740p^3$ also has a local minimum in the relevant range. Similarly, $g''(1/2) < 0$ and $g''(1) > 0$, such that $g''(p)$ also crosses the abscissa once from below, implying one local minimum of $g'(p) = 21 - 128p + 333p^2 - 408p^3 + 185p^4$. As both $g'(1/2)$ and $g'(1)$ are positive but there are negative values of $g'(p)$ in between, the first derivative first crosses the abscissa from above and then again from below. Hence, $g(p)$ first has a local maximum and then a local minimum. As $g(1/2)$ is positive, the local maximum must be in the positive domain, and because $g(1) = 0$ and $g'(1) > 0$, the graph crosses the abscissa from below at $p = 1$ and hence the local minimum is in the negative domain. This implies that there must be a root in between. It can be shown that this root lies at

$$\ddot{p} = \frac{65}{148} + \frac{1}{148\sqrt{\frac{3}{-941-9176\left(\frac{2}{8561+9\sqrt{916593}}\right)^{1/3}+74\times 2^{2/3}\left(8561+9\sqrt{916593}\right)^{1/3}}}}$$
$$+\frac{1}{2}\left[-\frac{941}{8214} + \frac{62}{111}\left(\frac{2}{8561+9\sqrt{916593}}\right)^{1/3} - \frac{1}{111}\left(\frac{1}{2}\left(8561+9\sqrt{916593}\right)\right)^{1/3}\right.$$
$$\left. + \frac{29241\sqrt{\frac{3}{-941-9176\left(2/(8561+9\sqrt{916593})\right)^{1/3}+74\times 2^{2/3}\left(8561+9\sqrt{916593}\right)^{1/3}}}}{2738}\right] \approx 0.81216.$$

Therefore, $\overline{w}_2$ is smaller than $\overline{w}$ for $1/2 < p < \ddot{p}$ and there are levels of w between $\overline{w}_2$ and $\overline{w}$ for which not searching is the unique equilibrium. $\square$

Proof of Lemma 5. The regulator's probability of winning contingent on the number of searches is given by

$$\Pr(d = -1|0) = 0$$
$$\Pr(d = -1|1) = (1-p)^2 p$$
$$\Pr(d = -1|2) = 2p(1-p)p^2 + (1-p)^2\left(2p(1-p) + p^2\right).$$

Using the respective probabilities and rearranging (7) and (8) gives

$$\frac{c}{w} \leq p - 3p^2 + 5p^3 - 3p^4 \tag{18}$$

and

$$\frac{c}{w} \leq p - \frac{5}{2}p^2 + 3p^3 - \frac{3}{2}p^4, \tag{19}$$

respectively. (7) is the relevant constraint if the RHS of (18) is smaller than the RHS of (19) or $p^2 g(p) \geq 0$ where $g(p) = 1/2 - 2p + (3/2)p^2$. The first derivative $g'(p) = -2 + 3p$ crosses the abscissa once from below and hence, $g(p)$ is convex. It is easy to verify that $g(p)$ has roots at $p^* = 1/3$ and $p = 1$ and hence is positive for values of p below p^* and negative for values above. Thus, (7) is binding for small p while (8) is relevant for large p and the lemma follows. $\square$

Proof of Proposition 1. We have to show that $\overline{w} > \underline{w}$ for all p. For $0 < p < \tilde{p}$, the relevant comparison is

$$\overline{w} = \frac{c}{p - 5p^2 + 16p^3 - 28p^4 + 26p^5 - 10p^6} \geq \frac{c}{p - 3p^2 + 5p^3 - 3p^4} = \underline{w}$$

or $2 - 11p + 25p^2 - 26p^3 + 10p^4 \geq 0$. Define the LHS as $g(p)$. The third derivative $g'''(p) = -156 + 240p$ is increasing and negative in the relevant range as $g'''(\tilde{p}) < 0$. This implies that $g''(p) = 50 - 156p + 120p^2$ is decreasing, and it is positive as its value at $\tilde{p}$ is positive. Hence, $g'(p) = -11 + 50p - 78p^2 + 40p^3$ is increasing, and from $g'(\tilde{p}) < 0$ we know that it is negative. Due to this fact, $g(p)$ is decreasing, and as $g(\tilde{p}) > 0$, it is positive and the condition above holds.

For $\tilde{p} < p < 1/3$, the relevant comparison is

$$\overline{w} = \frac{c}{p - 4p^2 + \frac{28}{3}p^3 - 13p^4 + 10p^5 - \frac{10}{3}p^6} \geq \frac{c}{p - 3p^2 + 5p^3 - 3p^4} = \underline{w}$$

or $3 - 13p + 30p^2 - 30p^3 + 10p^4 \geq 0$. Define the LHS as $g(p)$. The third derivative $g'''(p) = -180 + 240p$ is increasing and negative in the range of interest as $g'''(1/3) < 0$. Hence, $g''(p) = 60 - 180p + 120p^2$ is decreasing and positive as $g''(1/3) > 0$. The slope of $g'(p) = -13 + 60p - 90p^2 + 40p^3$ thus is positive, and the first derivative is negative because $g'(1/3) < 0$. From this we know that $g(p)$ is decreasing. As $g(1/3) > 0$, the LHS is positive throughout the range of interest and the above condition is satisfied.

For $1/3 < p < 1$, the relevant comparison is

$$\overline{w} = \frac{c}{p - 4p^2 + \frac{28}{3}p^3 - 13p^4 + 10p^5 - \frac{10}{3}p^6} \geq \frac{c}{p - \frac{5}{2}p^2 + 3p^3 - \frac{3}{2}p^4} = \underline{w}$$

or $9-38p+69p^2-60p^3+20p^4 \geq 0$. Define the LHS as $g(p)$. The third derivative $g'''(p) = -360+480p$ is increasing and crosses the abscissa once from below as $g'''(1/3) < 0$ and $g'''(1) > 0$. Hence $g''(p) = 138-360p+240p^2$ is convex and has a local minimum at $p = 3/4$. As its value is positive at the local minimum it is positive throughout the range of interest. Therefore, $g'(p) = -38+138p-180p^2+80p^3$ is increasing and negative as $g'(1/3) < 0$ and $g'(1) = 0$. That being the case, the LHS decreases in p and is positive as $g(1/3) > 0$ and $g(1) = 0$. The above condition holds. $\square$

Proof of Proposition 2. First, we show that $\underline{w}$ is smaller than $\tilde{w}$ for $p < 1/3$. For $0 < p < \dot{p}$ where the relevant upper bound is $\overline{w}$, we check whether

$$\underline{w} = \frac{c}{p-3p^2+5p^3-3p^4} \leq \frac{c}{p-5p^2+16p^3-28p^4+26p^5-10p^6} = \overline{w}$$

or $2-11p+25p^2-26p^3+10p^4 \geq 0$. Define the LHS as $g(p)$. Observe that $g'''(p) = -156+240p$ is increasing and negative throughout the relevant range, which means that $g''(p) = 50-156p+120p^2$ is decreasing. As $g''(\dot{p}) > 0$, the second derivative is positive, which implies a positive slope for $g'(p) = -11+50p-78p^2+40p^3$. As $g'(\dot{p}) < 0$ we know that the first derivative is negative and by that we know that $g(p)$ is decreasing. The fact that $g(\dot{p}) > 0$ implies that the LHS is positive throughout the relevant range and that the conditions above holds.

For $\dot{p} < p < 1/3$, the relevant upper bound for the wage is $\underline{w}_1$ and hence, the relevant comparison is

$$\underline{w} = \frac{c}{p-3p^2+5p^3-3p^4} \leq \frac{c}{3p^2-6p^3+3p^4} = \underline{w}_1$$

or $1-6p+11p^2-6p^3 \geq 0$. Define the LHS as $g(p)$. The second derivative $g''(p) = 22-36p$ is decreasing and positive for $p = 1/3$ and thus positive in the relevant range so that $g'(p) = -6+22p-18p^2$ is increasing. As $g'(1/3) < 0$, the first derivative is negative which implies a negative slope of $g(p)$. Together with the facts that $g(\dot{p}) > 0$ and $g(1/3) = 0$ we know that $g(p)$ is positive in the relevant range and the condition above holds.

Next we show that for values of p above $1/3$, $\underline{w}$ is never below $\tilde{w}$. First, for $1/3 < p < 1/2$, the relevant comparison is

$$\underline{w} = \frac{c}{p - \frac{5}{2}p^2 + 3p^3 - \frac{3}{2}p^4} \geq \frac{c}{3p^2 - 6p^3 + 3p^4} = \underline{w}_1$$

or $-2 + 11p - 18p^2 + 9p^3 \geq 0$. Define the LHS as $g(p)$. The second derivative $g''(p) = -36 + 54p$ is increasing and negative for $p = 1/2$ and thus negative in the relevant range, implying a negative slope of $g'(p) = 11 - 36p + 27p^2$. As $g'(1/3) > 0$ and $g'(1/2) < 0$, there must be one root of the first derivative in the relevant range. This implies that $g(p)$ is concave and has a local maximum in that range, and because $g(1/3) = 0$ and $g(1/2) > 0$, it is positive throughout the range and the condition above holds. Second, for $1/2 < p < 1$, the relevant comparison is

$$\underline{w} = \frac{c}{p - \frac{5}{2}p^2 + 3p^3 - \frac{3}{2}p^4} \geq \frac{c}{3p^2 - \frac{15}{2}p^3 + \frac{15}{2}p^4 - 3p^5} = \underline{w}_2$$

or $-2 + 11p - 21p^2 + 18p^3 - 6p^4 \geq 0$. Define the LHS as $g(p)$. The third derivative $g'''(p) = 108 - 144p$ is decreasing and crosses the abscissa once because $g'''(1/2) > 0$ and $g'''(1) < 0$. This implies that $g''(p) = -42 + 108p - 72p^2$ is concave and has a local maximum at $p = 108/144$ in that range. As $g''(108/144) < 0$, the second derivative is negative throughout. The first derivative $g'(p) = 11 - 42p + 54p^2 - 24p^3$ hence is decreasing in the relevant range. As $g'(1/2) > 0$ and $g'(1) < 0$ the first derivative has one root and $g(p)$ is concave and has a local maximum. From $g(1/2) > 0$ and $g(1) = 0$ we can infer that $g(p)$ is positive in the relevant range and the condition above holds.

The second part of the proof follows from the earlier analysis. For $1/2 < p < \ddot{p}$, $\overline{w}_2 < \overline{w}$ from Lemma 4 and $\underline{w} < \overline{w}$ from Proposition 1. Hence, whenever $w > \max\{\overline{w}_2, \underline{w}\}$ it is below $\overline{w}$ and the unique equilibrium exists for this range of p. □

Proof of Proposition 3. There are three cases. For $0 < p < 1/3$, the losses in Situation 1 L_1 are larger than the losses in Situation 2 L_2 if

$$L_1 = 9p^3 - 21p^4 + 18p^5 - 6p^6 \geq p - 5p^2 + 14p^3 - 22p^4 + 18p^5 - 6p^6 = L_2$$

or $-1 + 5p - 5p^2 + p^3 \geq 0$. Define the LHS as $g(p)$. The second derivative $g''(p) = -10 + 6p$ is increasing and negative throughout the relevant range as $g''(1/3) < 0$.

Hence $g'(p) = 5 - 10p + 3p^2$ is decreasing, and it is positive because $g'(1/3) > 0$. It follows that $g(p)$ has a positive slope. As $g(0) < 0$ but $g(1/3) > 0$ it crosses the abscissa once from below in the range of interest. It can be shown that this root lies at $p = 2 - \sqrt{3}$, and hence the condition above holds for values of p larger than that.

For $1/3 < p < 2/3$, the relevant comparison is

$$L_1 = 3p - 15p^2 + 30p^3 - 30p^4 + 18p^5 - 6p^6 \geq p - 5p^2 + 14p^3 - 22p^4 + 18p^5 - 6p^6 = L_2$$

or $1 - 5p + 8p^2 - 4p^3 \geq 0$. Define the LHS as $g(p)$. The second derivative $g''(p) = 16 - 24p$ is decreasing and positive as $g''(2/3) = 0$. Thus, $g'(p) = -5 + 16p - 12p^2$ is increasing and crosses the abscissa once from below as $g'(1/3) < 0$ and $g'(2/3) > 0$. It can be shown that $g'(1/2) = 0$. From this we can infer that $g(p)$ is convex and has a local minimum. Its value at the local minimum is zero, so the condition above holds in the relevant range of p.

Lastly, for $2/3 < p < 1$, the relevant condition is

$$L_1 = 3p - 9p^2 + 15p^3 - 21p^4 + 18p^5 - 6p^6 \geq p - 5p^2 + 14p^3 - 22p^4 + 18p^5 - 6p^6 = L_2$$

or $2 - 4p + p^2 + p^3 \geq 0$. Define the LHS as $g(p)$. The second derivative $g''(p) = 2 + 6p$ is increasing and positive throughout the whole range of interest, implying a positive slope for $g'(p) = -4 + 2p + 3p^2$. As $g'(2/3) < 0$ and $g'(1) > 0$, the first derivative crosses the abscissa once from below. From this we can infer that $g(p)$ is convex and has a local minimum. We know that this local minimum is negative as $g(2/3) > 0$, $g(1) = 0$, and $g'(1) > 0$, and therefore, $g(p)$ crosses the abscissa once from above. It can be shown that this root lies at $p = \sqrt{3} - 1$, and the condition above holds for values of p smaller than that. $\square$

Supplementary Material B: Other pure-strategy equilibria

We show that equilibria in pure strategies exist in which the regulator searches once or twice given three searches by the firm.

We begin with the equilibrium where the regulator searches once. In this case, the judge decides according to the decision rule given in Table 4. If the expected value of information against the proposal on the two remaining dimensions where

	{0,0}	{1,0}	{2,0}	{3,0}	{0,1}	{1,1}	{2,1}	{3,1}
$0 < p \leq 1/2$	R	F	F	F	R	R	F	F
$1/2 < p \leq 1$	R	R	F	F	R	R	R	F

Table 4: Decision rule for $\mu_f = 3$ and $\mu_r = 1$. F (R) denotes $d = 1$ ($d = -1$).

the regulator did not search, $2 \times p^2 + 1 \times p(1-p) = 2p$, is larger than the value of evidence in favor of the proposal, then the decision is made against the proposal. In the two cases where the firm has one more piece of information than the firm ({1,0} and {2,1}), the expected value of contra information is large enough to tip the decision towards the regulator for $p > 1/2$. We again assume that when the regulator presents more than one piece of information out of equilibrium, the judge believes that the regulator did search in all other dimensions as well and did not discover any evidence. Hence, the decision rule under full information applies out of equilibrium.

The following conditions must hold such that searching once is optimal for the regulator, given three searches by the firm and the corresponding belief by the judge, $\mu_f = 3$, $\mu_r = 1$:

$$\Pr(d = -1|1)w - c \geq \Pr(d = -1|3)w - 3c \tag{20}$$

$$\Pr(d = -1|1)w - c \geq \Pr(d = -1|2)w - 2c \tag{21}$$

$$\Pr(d = -1|1)w - c \geq \Pr(d = -1|0)w. \tag{22}$$

We have the following result.

Lemma 6. *If and only if $\underline{w}_1 \leq w \leq \overline{w}_1$ and $0 < p < 1/2$, where $\underline{w}_1 = c/(3p^2 - 6p^3 + 3p^4)$ and $\overline{w}_1 = c/(3p^2 - 9p^3 + 9p^4 - 3p^5)$, there exists an equilibrium where the regulator searches once, the firm searches three times, and the judge has beliefs $\mu_f = 3$ and $\mu_r = 1$. For $1/2 \leq p < 1$, only one search by the regulator never is optimal.*

Proof. First, for $0 < p < 1/2$, the winning probabilities for the regulator are

$$\begin{aligned}
\Pr(d=-1|0) &= (1-p)^3 \\
\Pr(d=-1|1) &= (1-p)^3 + 3p(1-p)^2 p \\
\Pr(d=-1|2) &= (1-p)^3 + 3p(1-p)^2 \left(2p(1-p)+p^2\right) \\
\Pr(d=-1|3) &= (1-p)^3 + 3p(1-p)^2 \left(1-(1-p)^3\right) + 3p^2(1-p)p^3.
\end{aligned}$$

The incentive constraint (22), which ensures that one search is better than no search, gives the lower bound for the wage $\underline{w}_1 = c/(3p^2 - 6p^3 + 3p^4)$. For the moment, ignore (20). Condition (21), which ensures that one search is more profitable than two searches, gives the upper bound for the wage, $\overline{w}_1 = c/(3p^2 - 9p^3 + 9p^4 - 3p^5)$. The upper bound $\overline{w}_1$ is above the lower bound $\underline{w}_1$ if

$$\frac{c}{3p^2 - 6p^3 + 3p^4} < \frac{c}{3p^2 - 9p^3 + 9p^4 - 3p^5}$$

or $p^3(3 - (6p - 3p^2)) > 0$. The term $6p - 3p^2$ has its global maximum at $p = 1$ with value 3 and is strictly concave, hence the condition is satisfied for the relevant range of p, $0 < p < 1/2$. It remains to be checked that (20) is slack. Plugging $\overline{w}_1$ into (20) yields

$$\frac{-6p^2 + 21p^3 - 27p^4 + 12p^5}{3p^2 - 9p^3 + 9p^4 - 3p^5} c \geq -2c$$

or $p^3\left(1 - (3p - 2p^2)\right) \geq 0$. The term $3p - 2p^2$ is strictly concave and has its maximum at $p = 3/4$. It is strictly increasing in the relevant range $0 < p < 1/2$ and takes on value 1 at $p = 1/2$. Hence, the condition is satisfied for that range and (20) is slack.

For $1/2 \leq p < 1$, the winning probabilities for the regulator are

$$\begin{aligned}
\Pr(d=-1|0) &= (1-p)^3 + 3p(1-p)^2 \\
\Pr(d=-1|1) &= (1-p)^3 + 3p(1-p)^2 \\
\Pr(d=-1|2) &= (1-p)^3 + 3p(1-p)^2 + 3p^2(1-p)2p(1-p) \\
\Pr(d=-1|3) &= (1-p)^3 + 3p(1-p)^2 + 3p^2(1-p)(3p(1-p)^2 + p^3).
\end{aligned}$$

It is obvious that for any positive value of search cost c and non-negative benefit w, the constraint (22) can never hold. Hence, an equilibrium with one search effort by the regulator does not exist for $1/2 \leq p < 1$. $\square$

As the next step, we characterize an equilibrium where the regulator searches twice. The judge decides in favor of the party that delivers more pieces of information. In case of a tie ({0,0}, {1,1}, {2,2}) the decision is made in favor of the regulator as the expected value of contra information on the third dimension where no search has taken place is positive.[27] We again assume that when the regulator presents more than two pieces of information out of equilibrium, the judge believes that the regulator did search in the third dimension as well and did not discover any evidence. Hence, the decision rule under full information applies out of equilibrium.

The following conditions must hold such that two searches are optimal for the regulator, given three searches by the firm and the corresponding belief by the judge, $\mu_f = 3$, $\mu_r = 2$.

$$\Pr(d = -1|2)w - 2c \geq \Pr(d = -1|3)w - 3c \tag{23}$$

$$\Pr(d = -1|2)w - 2c \geq \Pr(d = -1|1)w - c \tag{24}$$

$$\Pr(d = -1|2)w - 2c \geq \Pr(d = -1|0)w \tag{25}$$

The three conditions guarantee that the regulator prefers two searches to three, one, and zero searches.

Lemma 7. *If and only if $\underline{w}_2 \leq w \leq \overline{w}_2$, where*

$$\underline{w}_2 = \begin{cases} c/\left(3p^2 - 9p^3 + 12p^4 - 6p^5\right) & \text{for } 0 < p < 1/2 \\ c/\left(3p^2 - (15/2)p^3 + (15/2)p^4 - 3p^5\right) & \text{for } 1/2 \leq p < 1, \end{cases}$$

and $\overline{w}_2 = c/\left(3p^2 - 12p^3 + 24p^4 - 24p^5 + 9p^6\right)$, there exists an equilibrium where the regulator searches twice, the firm searches in all three dimensions and the judge has beliefs $\mu_f = 3$ and $\mu_r = 2$.

[27] Note that when the judge receives the information {3,3} out of equilibrium, the decision is made for the firm.

Proof. The winning probabilities for the regulator are

$$\begin{aligned}
&\Pr(d=-1|0) = (1-p)^3\\
&\Pr(d=-1|1) = (1-p)^3 + 3p(1-p)^2 p\\
&\Pr(d=-1|2) = (1-p)^3 + 3p(1-p)^2(2p(1-p)+p^2) + 3p^2(1-p)p^2\\
&\Pr(d=-1|3) = (1-p)^3 + 3p(1-p)^2(1-(1-p)^3) + 3p^2(1-p)(3p^2(1-p)+p^3).
\end{aligned}$$

The upper bound for the wage $\overline{w}_2 = c/\left(3p^2 - 12p^3 + 24p^4 - 24p^5 + 9p^6\right)$ is given by condition (23). Conditions (24) and (25) can be written as

$$w \geq \frac{c}{3p^2 - 9p^3 + 12p^4 - 6p^5}$$

and

$$w \geq \frac{c}{3p^2 - (15/2)p^3 + (15/2)p^4 - 3p^5},$$

respectively. (24) is binding if

$$\frac{c}{3p^2 - 9p^3 + 12p^4 - 6p^5} < \frac{c}{3p^2 - (15/2)p^3 + (15/2)p^4 - 3p^5} \tag{26}$$

or $p^3\left(-\frac{3}{2} + \frac{9}{2}p - 3p^2\right) > 0$. The polynomial in parentheses is negative at $p = 1$ and equal to zero at $p = 1$. It has another root at $p = 1/2$ and hence is negative for values of p below $1/2$ and positive for values of p above. Taken together with the root at $p = 0$ from the term p^3, condition (26) does not hold for $0 < p \leq 1/2$ and holds for $1/2 < p < 1$. Hence, constraint (25) is binding for smaller p and (24) for larger p.

It remains to be shown that the upper bound $\overline{w}_2$ lies above the lower bound $\underline{w}_2$. For $0 < p \leq 1/2$, we need to check whether

$$\frac{c}{3p^2 - 9p^3 + 12p^4 - 6p^5} < \frac{c}{3p^2 - 12p^3 + 24p^4 - 24p^5 + 9p^6},$$

which is equivalent to $p^3(3 - 12p + 18p^2 - 9p^3) > 0$. Define the term in parentheses as $g(p) = 3 - 12p + 18p^2 - 9p^3$. The second derivative $g''(p) = 36 - 54p$ is positive for $p = 0$ and negative for $p = 1$ and has one root at $p = 2/3$. Hence, $g'(p) = -12 + 36p - 27p^2$ is strictly concave and takes on value 0 at its global maximum. The original function $g(p)$ is positive for $p = 0$ and zero for $p = 1$. It is non-increasing throughout $[0,1]$, convex up to the root of $g'(p)$ and concave thereafter. Hence it cannot have another root in the relevant range. Taken together, the condition is above satisfied and $\overline{w}_2$ lies above $\underline{w}_2$ for $0 < p \leq 1/2$. For $1/2 < p < 1$, the comparison is

$$\frac{c}{3p^2 - (15/2)p^3 + (15/2)p^4 - 3p^5} < \frac{c}{3p^2 - 12p^3 + 24p^4 - 24p^5 + 9p^6}$$

or $p^3(3 - 11p + 14p^2 - 6p^3) > 0$. Let $g(p) = 3 - 11p + 14p^2 - 6p^3$. The second derivative $g''(p) = 36 - 54p$ crosses the abscissa once from above. Hence, $g'(p) = -12 + 36p - 27p^2$ is concave and has its global maximum at $p = 7/9$ with a value of $-1/3$. Thus, the original function $g(p)$ is falling in the interval $[0,1]$, is positive at $p = 0$ and equal to 0 at $p = 1$, and therefore cannot have another root in that interval. Taken together, the condition above is also satisfied for values of p between $1/2$ and 1. $\square$